The Sextant Equation

$E = mc^2$

Peter Galison

On 17 November 1945, John Wheeler, Princeton physicist, Manhattan Project veteran and herald of a new age of physics, stood before a symposium audience to survey the state of his science. He began by recalling that first moment of the nuclear age, early in the war, at the University of Chicago. A key figure in the war effort had telephoned Washington to tell the president of Harvard and head of the National Scientific Research Board, James Conant, about the events that the refugee physicist Enrico Fermi had just directed: 'The Italian navigator has discovered America.' 'Splendid,' Conant replied, 'and is the new country safe to enter?' The report: 'Yes and Columbus finds the natives are friendly.' That was 2 December 1942, and the coded discussion told Conant and those responsible for the American scientific war effort that the world's first nuclear reactor had safely begun a self-sustaining chain reaction. Physicists had landed on the continent of applied nuclear fission where they could begin to imagine producing power or detonations from energy buried in the heart of the uranium atom. Over the following thirty-two months the scientists of the atomic-bomb project drove relentlessly towards the delivery of nuclear arms, ending, or rather pausing, in the cataclysmic blasts over Hiroshima and Nagasaki in August 1945.

Now, as Wheeler was speaking, just three months had passed since the war's end. Physics, not long before a relatively obscure academic redoubt, now lay front and centre in the nation's attention. Surveying physics and the society around it, Wheeler had a vision of the 'formation of the new world' augured by nuclear physics, 'the great continent which lies beyond [fission] and which represents the last untraversed portion of knowledge of the physical universe'. Mathematicians at the time of Columbus, Wheeler commented, could delude themselves about just how far the explorer had reached in his quest to circumnavigate the globe. Physicists of the mid-1940s, by contrast, could not deceive themselves about what remained to be found. For scientists now held in their hands a sextant of a simplicity that left no room for self-deception. Such a theoretical instrument, such a measure of scientific progress, would at any given moment tell the human race just how far it had progressed towards the total annihilation of matter into energy. Powerful as it was, uranium fission took humankind but a thousandth of the way towards the goal of total energy conversion, for only a thousandth of the mass of a uranium atom blew into pure energy when the uranium nucleus split. A pure swap of matter into energy would, by contrast, provide the final limit to energy production, the ultimately efficient production of energy that could be used to construct a new industrial world. Or to provide a weapon of unequalled power. And the sextant of modern science giving the measure of success, showing humankind its precise location on the scale of total conversion, was Albert Einstein's $E = mc^2$, the most famous equation in the history of science.

What this means is that if a mass of m grams is lost in the splitting of a uranium atom (the parts weigh less than the whole), then the amount of energy released in that fission process is E (in ergs), where E is given by the mass times the speed of light in a vacuum (30 billion centimetres per second) squared. Surprisingly enough, in Einstein's first paper he did not use E for *Energie* or Energia in German and Greek respectively, and c for *celeritas* (swift in Latin), but rather L for energy (surely after *lebendige Kraft* 'living' or kinetic energy) and V for the velocity of light. Although the particular symbols of $E = mc^2$ feel inevitable to us now that we have grown used to those particular symbols, Einstein settled on the E and c only in 1912. Energy can be released in various forms – in the simplest possible version of nuclear fission, a uranium atom divides into two smaller nuclei flying away from each other at a furious rate. The energy released by the splitting of a single uranium atom would be enough to budge a grain of sand visibly off a table; releasing the fission energy contained in the million billion billion atoms of a kilogram of uranium would – and did – destroy several square miles of city.

By the end of 1945, fission, the physics of nuclear reactors and atomic bombs, still held open questions, but it was, in large measure, an understood science. Beyond the cascading neutrons of the fission chain reaction (neutrons splitting nuclei in such a way that more neutrons emerged to split other nuclei that in turn made more neutrons . . .), however, still lay a panoply of problems entirely outside the physicist's command. How do neutrons and protons produce new particles through collisions? Startling new information about these novel processes arrived every month from experimenters' observations of the cosmic rays, mostly protons, that rained down on the Earth's upper atmosphere from deep space. Again Wheeler: 'The possibility of the complete conversion of matter to energy is suggested by present incomplete information on the production of particles of lower mass by or from protons in the upper atmosphere of the Earth.' Wheeler dreamed of a process that would convert *all* of a piece of matter into energy.

Grasping the nature of these particle transformations fascinated Wheeler and his contemporaries. Soon he was launching teams of physicists on quests high in the atmosphere using bombers just back from the war front; Wheeler joined captured German scientists at the White Sands Proving Grounds where they fired unmanned V-2 missiles laden with instruments over a hundred miles into the threshold of space. Glimpses of high-energy particles from deep space beckoned – there was physics to be had up there, but the particles were too rare to be the basis for a full-scale campaign of physical research. What was needed was a consistent and copious source of energetic particles – in this respect, deep space could not compete with the campaign to build larger and more powerful particle accelerators. Needed too would be observations that would record the changes induced in bits of matter when struck by high-energy particles. And finally, physicists would have to produce a new, consistent theory that would capture the relations between elementary particles and the forces that governed their interaction.

The sextant equation $E = mc^2$, according to Wheeler, would guide physicists as they manipulated accelerators, cosmic rays and theories towards the creation of a new field of science: elementary-particle physics. So it has – over the next decades particle accelerators pounded stationary targets with ever-faster projectiles and then shifted towards ramming particles into their antiparticles. Electrons slammed into positrons and protons into antiprotons, each cutting-edge accelerator upping the amount of energy produced and pushing further into the physics of the very small. From the late 1940s into the first decade of the twenty-first century, that burgeoning domain of accelerator-based physics used energy-mass conversion to call

into observable existence the basic constituents of matter. Starting with the proton, the neutron, the electron and the positron, the particle zoo's population proliferated as physicists used the energy produced in collisions to create new and different kinds of particles. Already in 1932, the positron, antiparticle to the electron, appeared in an experimentalist's vessel, showing dramatically that matter and antimatter annihilate each other to produce pure energy, and conversely that pure energy could produce a particle and its antiparticle twin.

In the decades after World War II, it became possible to produce and then to manipulate particles like the pion that were of intermediate mass between the proton and the electron. Heavier, excited versions of protons and neutrons issued from the collisions of protons and mesons on nuclei – and the menagerie grew. When electrons and anti-electrons, pions and antipions, or protons and antiprotons could be skilfully battered against one another, their annihilation was complete and the totality of their conjoint energy became available for the production of new subatomic entities. Over the 1960s and 1970s, these pairs of particles were joined by subnuclear quark-antiquark pairs in their various combinations, along with heavier versions of the electron and new force-carrying particles to form the 'standard model' of particle physics. It is the equation $E = mc^2$ that lies behind the enormous accelerators that for three decades have driven particles into their antiparticles. Directly out of these colliding-beam facilities came the canonical formulation of particle physics of the 1970s. It has since remained essentially intact.

In those months towards the end of World War II, the interconvertibility of energy and mass held limitless promise and threat. In June 1945, Wheeler mused, 'Discovery of how to release the untapped energy on a reasonable scale might completely alter our economy and the basis of our military security. For this reason we owe special attention to the branches of ultra-nucleonics [physics beyond the by then rather well-understood physics of nucleons, that is neutrons and protons].' That more distant field would embrace new physics not seen in the wartime laboratories: cosmic-ray phenomena, meson-physics field theory, energy production in supernovae, and particle-transformation physics. Abstract inquiry into physics that probed below the scale of the nuclear, according to Wheeler, would clearly fuse with the 'country's war power'. For Wheeler knew perfectly well that among the tasks of 'ultra-nucleonics' lay the possibility of a more complete use of the energy proclaimed by $E = mc^2$ than that tiny thousandth part liberated by nuclear fission.

Fission's only partial release of energy meant that Hiroshima had been destroyed by the conversion of mass weighing considerably less than the eraser at the top of a pencil. Such thoughts had led Wheeler – and many other physicists – to wonder whether the sextant equation might point the way towards a much more complete release of energy.

Before the site of the Los Alamos weapons laboratory was anything but a country boys' school, a small group of other nuclear illuminati gathered at Berkeley to discuss nuclear weapons. J. Robert Oppenheimer was there as America's foremost quantum theorist. So was Hans Bethe, the physicist who, before he fled 1930s Germany, had figured out the nuclear physics that explained why the sun shines. They were joined by a stellar group, including the Hungarian refugee Edward Teller, later known as the 'father of the H-bomb'. In the brash hothouse environment of those early days, fission weapons seemed trivial to them: ram enough fissile uranium together and it would detonate. They assigned the problem to a young Berkeley physicist, Robert Serber, arrogating for themselves an infinitely more subtle and challenging problem: the hydrogen bomb, or H-bomb. The H-bomb would work by forcing together low-mass nuclei such as those of hydrogen, rather than prying apart heavy nuclei like uranium. But as the high-mesa laboratory of Los Alamos began to take shape, it became ever more obvious that building an A-bomb was anything but trivial. Project leaders, including Oppenheimer and the German refugee Hans Bethe, shunted the H-bomb aside in order to produce a usable weapon by war's end. Edward Teller, however, held tenaciously to the idea and, over the course of the war, moved determinedly away from the mainline fission work towards the defence and development of the weapon that gripped his imagination.

On 12 August 1945 Wheeler, then on the Pacific island of Tinian, which served as a staging area for the nuclear strikes, penned a letter to Teller: 'Dear Edward, with the conclusion of the war today my work here will soon reach its conclusion . . . What I can now do most effectively is, I believe, fundamental research. But I do not feel quite at ease to do so over the next five-year period.' He recalled Teller's previous invitation to work on the fusion weapon, and his own conviction that the H-bomb was a weapon destined for the next war, not the present campaign against the Axis. With the Japanese surrender, contemplation of that next conflict had for Wheeler become inevitable – he expected war with the Soviets to occur in the very near future. And that would be a conflict in which fusion, converting a higher proportion of matter into energy (according to $E = mc^2$), would be vital. For security reasons Wheeler went metaphorical:

> Here is a group of men absolutely isolated on an island. They have got into a fight. Two groups of men with quite different ways of doing things have teamed together to try to put down the troublemakers. Our group has learned to put together a bow and arrow. By that means we have put an end to the fighting. Our ally is observant. Now that the fight is over he has gone back and is spending part of the time behind his wall. We know that some of his men would get delight out of building a bow and arrow of their own. We suspect that some of his men would not hesitate to use that bow and arrow on us if someday we happened to get into a disagreement on who is to get the pears from that fine-looking pear tree over there. For some reason or other the two former allies don't seem to be able to get together to turn the bow and arrow weapon over to a custodian whom both can trust . . . Some people in our group say, 'So what' and are making plans to go fishing. I'm one of the people who feels that if we're going to get into an armament race, we'd better start now, and we'd better try to build the best weapon we know how to build – a machine gun which will outmode the bow and arrow.

Wheeler ended by saying that he thought he had better begin thinking about the 'machine gun' if conflict might break out over the next five to ten years. He did, launching a major Princeton-based H-bomb design effort known as 'Matterhorn B' alongside his more pacific inquiries into the transformations of mass into energy. In fact, just next door to Wheeler's part of the Matterhorn project stood another – with the aim of producing energy for civilian use through nuclear fusion. But with nuclear fusion, a thousand times more energy would be released per nuclear collision than in fission. Suddenly one could imagine that bombs the physical size of the ones used against Hiroshima and Nagasaki might deliver the explosive equivalent of 10–20 million tons of TNT, not the 10,000–20,000 tons of TNT equivalent to the World War II atomic bombs. And in principle one could picture making bombs of unlimited destructive capability – within a few years people began to discuss the production of gigaton hydrogen bombs that would blow a hole all the way through the atmosphere. Throughout, Wheeler saw the sextant equation as a compass that would give coordinates on a map leading towards the total conversion of matter into energy, for weapons and for wisdom. For despite its awful destructive force, even the hydrogen bomb still left much of the original mass unconverted to energy.

One peaceable exploration led Wheeler to imagine a new kind of atom: an electron and a positron orbiting one another. After a mere ten-billionth of a second, the new 'positronium' atom would decay as the two partners fell into each other in mutual annihilation, releasing their energy as two photons. Here was a beautiful, pure example of $E = mc^2$: if the positron and electron each had mass m, then the energy released would be $2\,mc^2$ and the two photons would each have a frequency f given by the equation $hf = mc^2$ (since, as Einstein had shown back in 1905, the photon's energy is $E = hf$.) One could look for those energetic photons that flew off back to back. Not too long afterwards physicists at MIT found them in an experiment, bang on the frequency where Wheeler – using Einstein's equation – said they would be.

From there a myriad of other transformations seemed to beckon. Nucleons smacked into each other in cloud chambers (vats of water vapour that made visible the tracks of particles) and yielded a host of new entities. How did these 'nuclear explosions' increase in probability with the energy of the incoming particles? What characterized the kind and number of explosion products? As far as Wheeler and many of his colleagues were concerned, answering questions like these would take the world of physics ever closer to a full understanding of their marvellous sextant $E = mc^2$.

Fission, fusion, positronium, accelerators, cosmic rays, black-hole dynamics – so much of late-twentieth-century physics ties back to that simply stated equation. But its origins lay far from the big physics of laboratories like Fermilab outside Chicago or CERN on the Swiss-French border, far from the weapons laboratories of Los Alamos, Livermore – or Arzamas-16. Young Einstein could scarcely have foreseen these developments when he first wrote down the equation.

We must go back to Einstein's world, the world surrounding him as he stood, as a clerk, in the patent office of 1905. This was a world in which electrification was a central pillar of modernization. Construction crews were ripping streets apart to build electric tramways, electricians were tearing gas lamps from ceilings and walls to make room for electric lighting. Crisscrossing their way across the United States, Europe and Russia, industrial power companies spun a great web of power lines, electric generators and measuring devices so they could deliver power to factories, cities and dwellings. Einstein's own family – his father and uncle – ran a fairly typical small electrotechnical business where they manufactured clocklike devices to measure power and other electrical quantities. Maxwell's equations, now taught in every elementary physics class, were in the 1870s still new enough to be only incompletely taught even in advanced schools, and Einstein clearly found the new theories and devices to be as fascinating as anything in science. The Bern patent office hired the twenty-three-year-old Einstein specifically to handle electrotechnical innovations – it was his job to assess their degree of novelty, to isolate and articulate the principles by which they worked.

It was from that Bern patent office, in his *annus mirabilis* of 1905, that Einstein published five extraordinary papers. Einstein's first article arrived at the journal *Annalen der Physik* on 18 March, displaying his theory of the light quantum; it was, in many ways, the paper that launched quantum physics. Six weeks later the young physicist submitted his doctoral dissertation, where he showed how to estimate the size of molecules by reasoning about the way in which big molecules like sugar contributed to the viscosity of sugar water. On 11 May Einstein submitted his account of Brownian motion, demonstrating the effect of physically real atoms and molecules pounding on small suspended particles – think of smoke dust diffusing through air. It was a powerful intervention for the reality of atoms – atoms, in Einstein's reckoning, were not merely helpful fictions to be used in reckoning chemical processes. They were physical objects statistically slamming into the suspended particle, bit by bit driving it around the liquid.

Our concern here is with Einstein's fourth and fifth papers, submitted at the end of June and September. For it was in those two papers, no doubt his most famous ones, that Einstein introduced the special theory of relativity and derived as a consequence the famous equation of energy and mass. The relativity piece itself, 'On the electrodynamics of moving bodies', built on two simply stated starting principles and moved towards predictions from there. Avoiding detailed assumptions about this or that feature of the way particular objects were built or interacted, Einstein's theory hardly resembled the work of senior physicists of the time. Instead, it had an outsider's style – or perhaps a return to an older form of clarity.

As in his ideal physical theory, ~~thermo~~dynamics, Einstein wanted above all to start with *principles*. In thermodynamics, all rests on the twin pillars of the conservation of energy and the ever-increasing entropy of the world. In relativity, Einstein had in mind two other founding principles. First, Einstein asserted, the old starting point of classical physics would hold good for electricity and magnetism as well. That is, physicists since Galileo had accepted the proposition that one could not use mechanical means to tell whether one was 'really' moving if one was in a constantly moving enclosed box. (Galileo imagined the observer to be below deck in a sailing boat cutting evenly across the open sea; Einstein, not surprisingly, chose the train, sliding along smooth steel tracks, as the site of his thought experiments.) Einstein's insistent message was that Galileo still spoke to us. In the windowless hold of an evenly moving ship you couldn't watch a fish swim in a fish tank or drop a ball or conduct any mechanical experiment that would reveal your 'true' motion. So, Einstein added, would it be impossible to conduct any experiment in a smoothly running train with electricity, magnetism or light that would reveal that one was 'truly at rest'. This is the relativity principle.

Einstein's second starting point was, he confessed, at first quite surprising: within an inertial reference frame (one not accelerated), light travels at the same speed independently of the velocity of its source. Sit in a railroad station and measure the speed of light from a lantern fixed on the front of a stationary train engine – its light emerges at 186,000 miles per second. Now imagine a train hurtling through the station at half the speed of light, 93,000 miles per second. In ordinary classical physics a ball thrown from the moving train (in the direction that the train is moving) would sail by the station at the speed of the train *plus* the speed the thrower gave the ball. Astonishingly, Einstein says, this is not so for light. Sitting in the station, you would see the lantern light from the front of the high-speed train travel by you at 186,000 miles per second – and not a bit faster. Moreover, applying the first (relativity) principle, if you ran after a light beam shining away from you, you would never even begin to catch up. Regardless of inertial reference frame, regardless of the speed of the source, light always will be measured to be travelling at the same speed that we abbreviate by c. This is the second principle: the absolute speed of light.

From these two simply stated propositions, the physical equivalence of inertial reference frames and the absolute nature of the speed of light, Einstein changed physics for ever. In the process he overturned notions of space and time that had been the foundation of physical understanding since the time of Newton. Completing this work in May 1905, he began shortly afterwards to reflect on some consequences of the new physics. From Bern, he wrote to his friend Conrad Habicht on a summer Friday of 1905:

> I would love to have you here. You would soon become your old mischievous self again. – The value of my time does not weigh heavily these days; there aren't always subjects that are ripe for rumination. At least none that are really exciting . . . A consequence of the study of electrodynamics did cross my mind. Namely, the relativity principle, in association with Maxwell's fundamental equations, requires that the mass be a direct measure of the energy contained in a body; light carries mass with it. A noticeable reduction of mass would have to take place in the case of radium. The consideration is amusing and seductive; but for all I know God Almighty might be laughing at the whole matter and might have been leading me around by the nose.

Evidently persuaded that he was not causing God to laugh, Einstein wrote up his three-page $E = mc^2$ paper, 'Does the inertia of a body depend upon its energy content?', in September 1905; *Annalen der Physik* received it on the 27th of the month.

Before Einstein, there was already a great deal of discussion of how electromagnetic energy might be related to mass. In fact, some of the leading physicists of the day aimed to explain the existence of all inertial mass (the resistance of matter to being set in motion) as nothing other than the fact that charged particles, reacting to their own electric and magnetic fields, were hard to accelerate. Einstein himself never subscribed to such a reductionist programme; that is, one that aimed to show that everything, even inertia, was at root nothing but charge and electric and magnetic fields. It was also well established that a container of electromagnetic energy (a mirrored box filled with light, for example) would have a mass that rose in proportion to the electromagnetic energy that it held.

But Einstein was after far bigger fish – not content with an analysis of light, he was arguing that *any* form of energy had inertial mass associated with it. Not surprisingly, his $E = mc^2$ paper triggered debate. One of Einstein's allies, Max Planck – one of the leaders of German theoretical physics – lost little time in pointing out that a transfer of heat also adds mass. So, it seemed, a hot frying pan would weigh more than an otherwise identical cold one. This was new: nothing in Newtonian physics led one to expect that mass could possibly vary as a result of energy alone.

When Johannes Stark, a well-known senior physicist who later became an ardent Nazi, saw Planck's and Einstein's results, he attributed the discovery of the equivalence to Planck. That was too much for the young Einstein (who had not yet developed his Delphic style): 'I find it somewhat strange that you do not recognize my priority regarding the connection between inertial mass and energy.' Stark backed down quickly: 'You are greatly mistaken, esteemed colleague, if you think that I have not been doing sufficient justice to your papers. I champion you wherever I can, and it is my wish to be given the opportunity to propose you for a theoretical professorship in Germany quite soon.' To which a mollified Einstein replied, with regrets, that he had let 'a petty impulse goad me into making that remark about priority. . . . People who have been granted the privilege of contributing to the progress of science should not let their pleasure in the fruits of joint labour be spoiled by such things.'

Over the years following 1905, Einstein worked hard to generalize the result – to show that the equivalence of energy and mass was truly complete. Always pressed to come back to the equation, he offered three ways of deriving his best-known result. In the first, the original paper of 1905, Einstein imagined a body that emitted an equal burst of light back to back. Then he recalled how from the special theory of relativity he could look at the same situation from a different, unaccelerated reference system. Combining the two results he could deduce $E = mc^2$ but to show this properly one has to see precisely how energy transforms from one frame to another. Some twenty-nine years later, in a Pittsburgh lecture, Einstein presented a different argument for $E = mc^2$, this time using the fact that energy and momentum should be conserved in all inertial frames of reference. But it was his third, single-page argument that was the simplest: in 1946 he produced an $E = mc^2$ argument for the *Technion Journal* that required nothing from relativity theory but the basic assumptions. Let's take that last method and pause to consider Einstein's reasoning.

Suppose, as Einstein suggests, that one accepts four principles:

1) That the principle of special relativity holds good: that is, all reference frames that are not accelerated are equivalent. No one frame is 'truly' at rest, for example, only *relative* motions can be discussed in a physically meaningful way.

2) That momentum is conserved – after all, a fundamental article of faith even in classical physics. For ordinary matter, momentum is equal to mass multiplied by velocity. Conservation of momentum is the principle that if one adds up all the momentum, for example, for all the billiard balls on a table before they collide with one another, the same amount of momentum will be present after the collision.

3) That radiation has momentum – an experimentally tested result long accepted. (It is, for example, sunlight that pushes the tails of comets away from the Sun.)

4) That a moving observer sees a source of light as undergoing a change in its apparent angle ('stellar aberration'). In other words, it had long been known that an observer on Earth, for example, sees starlight as coming from a position shifted by a small angle α from the star's true place in the heavens. That angle depended on the velocity of the Earth, v, and was generally accepted to be, for velocities that were small compared with the speed of light, c, approximately $\alpha = v/c$. (This effect is easy to understand. If rain is falling straight down towards the ground and you run through it, you experience the rain as driving towards you at a certain angle. The faster you run, the bigger the 'aberration' of the rain from straight down; the angle would depend on the ratio of your speed to the speed of the rain. If, as you ran, you had a 'telescope' consisting of a long cardboard tube, you would have to angle it away from the vertical to have the raindrops fall straight through the tube. Similarly, because of the Earth's motion, optical telescopes need to be angled from a 'true' star position to see that star's light.

Suppose too, Einstein added, that we have one reference frame, the 'rest frame', which we might anachronistically identify as the frame of a space shuttle that is floating, engines-off, in deep space far from any objects like stars or planets, that would exert significant gravitational forces on it (Figure 1). In this frame a book hovers without moving in the middle of the shuttle, before two flashlights spaced equally on opposite sides each simultaneously flash a burst of light of energy *E/2* directly towards the book. The energy of both flashes is then absorbed by the book, so its energy increases by *E*. In the 'rest' reference frame of the space shuttle, the book doesn't go anywhere because it has been hit with equal impact by the light flashes coming from opposite directions.

Now, Einstein continued, let's look at exactly the same process from a different, 'moving' frame of reference (a Russian spaceship, say) moving

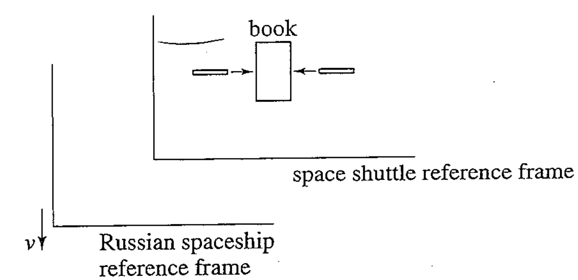

Figure 1

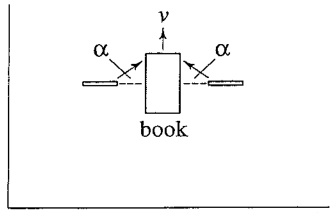

Figure 2

steadily downwards with velocity v. Viewed from this frame the scene looks like slightly different. As observed from the Russian spaceship, before our worthy book is hit by the twin flashes of light, the book will be moving *upwards* with velocity v (Figure 2). This means that in the spaceship frame, before the light beams hit the book, of mass M, the book's momentum is just Mv. The classical theory of light tells us that the momentum of a light burst of energy $E/2$ is just $E/2c$. Now in the Russian spaceship frame, the flashes appear not to travel horizontally but (because of the aberration effect) to arrive at a small angle, $\alpha = v/c$ to the horizontal.

In the Russian spaceship frame, the book's momentum *after* it is hit by the flashes is the sum of the original upward momentum of the book (Mv) and the momentum the book gets from the two light flashes, which in the Russian spaceship frame hit it at this 'aberration' angle.[1] Consequently, the light beams contribute a momentum Ev/c^2 to the book, which already had a momentum Mv: so the total momentum of the book in Russian spaceship frame after the absorption is $Mv + Ev/c^2$.

Although the book's momentum has increased, its final upward velocity is still v, the opposite of the Russian spaceship's velocity. (The book's velocity must remain v in the Russian spaceship frame: in the space shuttle frame, the light flashes hit the book in opposite directions and so leave it stationary; therefore, even after absorption, the book is still moving at v in the Russian spaceship frame.) So, as Einstein realized, the energy absorption must have increased the mass of the book – because the book's velocity does not increase, this is the *only* way of accounting for the increase in its momentum. If we denote the final mass of the book by M', then, in the frame of reference of the Russian spaceship:

$$\text{Final momentum of the book} = Mv + Ev/c^2 = M'v.$$

Dividing v from this equation and then subtracting M from both sides yields: $M' - M = E/c^2$, which is just another way of saying $E = (M' - M)c^2$. Now $M' - M$, the difference of the mass of the book before and after the arrival of the light flashes, can be abbreviated by the mass gained, m, giving the object of our desire,

$$E = mc^2$$

Now since one form of energy can always be converted into another, this result is not simply about light beams. It means that any form of energy adds to the inertial mass: a hot billiard ball is more massive than a cold one, and a spinning planet has more mass than a still one. In fact, if mass is allowed to turn into energy, it will. What might stand in its way? Conservation laws can – a conservation law is a statement that certain quantities don't change in a closed system. For example, you can't create electrical charge out of nothing. Or momentum – the tendency of a body to stay in straight motion once it is in motion – remains the same unless you apply a force to it. Because of these conservation laws, a single electron can't, in relativity theory, simply vanish into pure energy – that would strike electrical charge from the universe. Now if an electron hits an *anti*-electron (which has the opposite charge from that of an electron), the story is very different. Then the sum of their charges is zero (plus one added to minus one) so it *is* possible for the mass of the electron and positron to be converted completely to energy. Conversely, if the conservation laws are obeyed, pure energy can turn into mass – such as a positron and an electron.

Over the decades following 1905 $E = mc^2$ came to the laboratory. In 1932, two physicists from the famous Cavendish Laboratory in Cambridge, England, the experimentalists John Cockroft and Ernest Walton, showed that they could accelerate protons to bust apart a lithium nucleus. The resulting fragments of the lithium nucleus, it turned out, weighed less than the original lithium nucleus. At first it seemed as if mass had simply vanished. But by measuring the total energy of the flying fragments, the Cantabrigians, using $E = mc^2$, could show that the energy 'lost' in the mass change as the lithium nucleus broke precisely accounted for the energy contained in the fast-moving pieces that shot out from the broken nucleus. Einstein's formula had struck again.

But the world-changing use of $E = mc^2$ came with the discovery that neutrons could cause nuclear fission in uranium. For years the physicist Lise Meitner had been working with the chemist Otto Hahn in the Kaiser

Wilhelm Institute for Chemistry.[2] There in the leafy Berlin suburb of Dahlem, the physicist and the chemist bombarded nuclei with neutrons, using chemistry to sort out the products. For several years not only they, but also others, including Enrico Fermi's group in Rome, had concluded that the reaction products they were seeing after bombardment were actually new elements beyond uranium on the periodic table. Such 'transuranics', as they were called, appeared sensational, perhaps the greatest discovery of the new radioalchemy. In the Berlin collaboration the two kinds of skills they brought to the laboratory complemented each other: Meitner was the physicist of the outfit, Hahn the chemist. But compatibility in the lab meant nothing as the Nazis closed in and Meitner, who was Jewish, found her fate hanging by a thread. Finally, having been smuggled out of Germany by train on 13 July 1938, Meitner set up a rather threadbare scientific life in Sweden, where she anxiously awaited news from her collaborators, while the world hovered on the edge of war.

In Berlin the lab results only grew more confusing to Hahn, who continued the experiments. He and Meitner had long ago grown used to seeing products from the collision that in some reactions behaved like elements much lighter than uranium. But that, Hahn and everyone else believed, was merely a chemical illusion, an impossibility – the elements must be near uranium on the periodic table. 'Breaking' a nucleus into much smaller parts was simply impossible. One could chip off a proton or an alpha particle (two protons bound together with two neutrons). But breaking a nucleus squarely into two seemed, as one physicist later put it, like exploding a house by throwing a ball through a window. If a reaction product looked like barium, for example, it was probably the chemically related radium. Then things got really odd, and late one December night in 1938, Hahn wrote Meitner:

> 19.12.38 Monday eve in the lab. Dear Lise! . . . It is now just 11 p.m.; at 11.45 Strassmann [their other collaborator] is coming back so that I can eventually go home. Actually there is something about the 'radium isotopes' that is so remarkable that for now we are telling only you. . . . Our Ra[dium] isotopes act like Ba[rium].

'So please,' Hahn implored, 'think about whether there is any possibility' that there might be a variety of barium that was much heavier than usual.

Hahn sent his paper to the publisher three days after writing to Meitner, concluding his article with the conflicted sentiment that his and

Strassmann's chemical and physical souls were at war. They saw what looked like familiar light elements, but this just could not be: 'As chemists . . . we should substitute the symbols [of light elements] for [the heavy elements we have been discussing]. As "nuclear chemists" fairly close to physics we cannot yet bring ourselves to take this step which contradicts all previous experience in nuclear physics.'

When the letter of 19 December reached her, Meitner and her nephew the physicist Otto Robert Frisch, who had also fled, set out in the snow for a walk and began to pry apart the puzzling epistle. What would happen, they began to wonder, if the uranium nucleus, when hit by a neutron, began to oscillate like a fat water droplet? Thinking of the nucleus as such a drop had been current for some years. Suppose, they continued, that the whole droplet was normally in a rather delicate equilibrium, its 92 protons repelling each other furiously, and yet the whole held together by short-range but powerful nuclear attraction of the 238 or so protons and neutrons for one another. Then it might be that the droplet would distend itself as it oscillated, perhaps to the point where it would resemble a viscous barbell with globes at each end joined by a slender nuclear bar. At such a point of distension, the mutual repulsion of the protons located in the two globes could be more than the short-range nuclear bonds could counteract. Suddenly, driven by the electric repulsion of the globes, the nucleus could cleave into two parts, its two globes hurtling away from one another with the repelling force of two roughly equal bags of 46 protons. Meitner calculated. Two lighter nuclei would weigh less than they would together. And that mass difference, converted into energy according to $E = mc^2$, would be enormous. She and her cousin knew what no one else in the world suspected: back in Dahlem there was nuclear fission.

Events moved fast. The Danish physicist Niels Bohr, considered by many of his colleagues to have been the father of quantum theory, hearing Meitner and Frisch's interpretation, immediately understood where all his previous reasoning had gone wrong. Wheeler, who took the boat to America with Bohr in 1939, joined him in the mid-Atlantic composition of a comprehensive theoretical analysis of fission. One question led to another, as the physics sensation of the split atom jumped from laboratory to headline. And the next, immediate issue was vital to an unstable world: would the miscellaneous neutrons splattered about when the nucleus divided cause additional fissions? Could the fission of uranium cause a chain reaction? If it could, the enormous energy released by fission would multiply geometrically. Within months several physicists began to suspect that the fission process might, in the not too distant future, lead to the construction of nuclear bombs. Several pleaded with Einstein to write that fateful letter of 2 August 1939 to President Roosevelt:

> In the course of the last four months it has been made probable – through the work of Joliot in France as well as Fermi and Szilard in America – that it may become possible to set up nuclear chain reactions in a large mass of uranium, by which vast amounts of power and large quantities of new radium-like elements would be generated. Now it appears almost certain that this could be achieved in the immediate future. At stake, however, was more than abstract energy production. This new phenomenon would also lead to the construction of bombs, and it is conceivable – though much less certain – that extremely powerful bombs of a new type may thus be constructed. A single bomb of this type, carried by boat or exploded in a port, might very well destroy the whole port together with some of the surrounding territory.

Contact, Einstein insisted, would be needed between the administration and physicists. Ominously, Germany had stopped the sale of uranium. An intermediary representing the scientists' position saw President Roosevelt on 1 October 1939, and the atomic advocates pursued their concerns with a more technical memorandum by the Hungarian refugee Leo Szilard, the discoverer of the nuclear chain reaction. By then the Nazis had invaded Poland, and the snowball began its crashing descent. Fears of a German nuclear bomb rose; Pearl Harbor was attacked; and not long after the initiative the British prepared a seed project for a nuclear weapon. US committees evolved into laboratories and laboratories into the largest factories – of any kind – that the world had ever seen. Some years later, thinking back on those days, Einstein mused on the morality of what he had helped set in motion, first with the speculative scribbles of a young patent officer, and then later as the most famous scientist in the world:

> I made one mistake in my life – when I signed that letter to President Roosevelt advocating that the atomic bomb should be built. But perhaps I can be forgiven for that because we all felt that there was a high probability that the Germans were working on this problem and they might succeed and use the atomic bomb to become the master race.

Indeed, when Einstein was pressed to explain why people could discover atoms but not the means to control them, he replied, 'That's simple, my friend: because politics is more difficult than physics.'

By the end of the war, when John Wheeler delivered the address with which I began, $E = mc^2$ was, for physicists, a sign of the atomic times – celebrated for ending a war forced on them and regretted for precipitating an arms race. It was at one and the same time a guide to the future and a memorial of what had gone wrong.

After World War II, $E = mc^2$ was everywhere: it had long since left the physicists' control. One small marketing firm named itself after the equation: "You have to work smarter, not harder,' their self-description added. 'With images of Albert Einstein throughout the office as the "mascot" for the business, it would be difficult to do otherwise.'[3] $E = mc^2$ is also the name of a soft drink, a teenage science camp in Texas and the banner for a consortium of school districts in New Jersey that aims to improve science teaching. It is the title of a French best-seller by Patrick Cauvin (*$E = mc^2$, mon amour*), a love story about two eleven-year-old geniuses who take flight to Venice. Not surprisingly, you can order up a two-by-three-foot poster of Einstein himself emblazoned with this equation.

You might not expect the equation to make a particularly good piece of music, but Big Audio Dynamite did and, as best I can tell, nearly a dozen other rock groups have titled songs after the equation. A film – distributed on video – also bears the equation as its title: 'An Oxford physics professor tries to take Einstein one step further while balancing the demands of his wife and girlfriend – all the makings of nuclear fission!' There are $E = mc^2$ Japanese graphics companies and French Internet systems, Arizonian study groups and art installations from several countries. It is everywhere: symbol of genius, sign of power, harbinger of destruction.

Perhaps we should not be surprised. Unlike any other equation of physics, $E = mc^2$ binds to the broader culture in four ways. First, the equation itself is compact, easy to write, and dramatic in its implications for the laboratory and for the world. Einstein's equation governing the gravitational field, on the other hand, is more or ~~less~~ unpronounceable to the average person: $R_{ab} - \frac{1}{2} R\, g_{ab} = -8\pi G\, T_{ab}$, *mon amour* doesn't quite have that commercial snap, and is, I would wager, rather harder to write into a rock-and-roll hit – though physicists might rightly complain that the equation governing general relativity merits greater veneration than the mass-energy equivalence.

Second, the equation $E = mc^2$ captured, at least partially, the extraordinary fascination the broader culture of the arts and humanities has had with relativity's modification of ideas of space and time. Even before relativity, the painter Claude Monet was already fascinated by issues of simultaneity, speed, time and the alteration of space. When physics offered a world of non-Euclidean space-time and a fusing of temporality and spatiality, those notions, or at least metaphorical analogues of them, fell on fertile ground.

Third, after the British astronomer Arthur Eddington's eclipse expedition of 1919 proclaimed that Einstein's theory had correctly predicted the bending of starlight, Einstein became a cult figure standing all at once (at least for his adoring fans) as individual genius, pre-war pacifist, post-war conciliator and moral exemplar. Misunderstood, vilified and then lionized beyond measure, Einstein became a symbol of hope for anyone doing anything against the grain. For his enemies he was, of course, the anti-hero: cosmopolitan, anti-nationalist, Jew, abstract theorist, democrat, cut off from the so-called intuitions of earth, blood and nation. Even before World War II, Einstein, and through him his most famous equation, stood for the mixture of philosophy, physics and modernity that alternately seduced and horrified the world around him.

With the long hot and then cold war stretching from 1939 to 1989, the equation came to stand for something else – nuclear weapons – encapsulating in its sparse symbols both power and knowledge. Here the 'sextant equation' gained a fourth meaning, because these weapons seemed to combine the most esoteric understanding with the most terrible destructiveness. The equation came to signify an almost mystical force, embodying instantaneous and apocalyptic death.

It is in the confluence of these various cultural currents that we find the lines of affect that cluster around this equation. At once philosophy and genial fantasy, practical physics and terrifying weapon, $E = mc^2$ has become metonymic of technical knowledge writ large. Our ambitions for science, our dreams of understanding and our nightmares of destruction find themselves packed into a few scribbles of the pen.