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The Suppressed Drawing: Paul Dirac's Hidden Geometry

Purest Soul

For most of the twentieth century, Paul Dirac stood as the theorist's theorist. Though less known to the general public than Albert Einstein, Niels Bohr, or Werner Heisenberg, for physicists Dirac was revered as the "theorist with the purest soul," as Bohr described him. Perhaps Bohr called him that because of Dirac's taciturn and solitary demeanor, perhaps because he maintained practically no interests outside physics and never feigned engagement with art, literature, music, or politics. Known for the fundamental equation that now bears his name—describing the relativistic electron—Dirac put quantum mechanics into a clear conceptual structure, explored the possibility of magnetic monopoles, generalized the mathematical concept of function, launched the field of quantum electrodynamics, and predicted the existence of antimatter.

In this paper I will explore the meaning of drawing for Dirac in his work. In the thirteen hundred or so pages of his published work between 1924 and World War II, aside from a few graphs and a diagram in a paper that he coauthored with an experimentalist, Dirac had practically no use at all for diagrams. He never used them publicly for calculation, and I know of only two, almost trivial, cases in which he even exploited a figure for pedagogical purposes. His elegant book on general relativity contained not a single figure; his famous textbook on quantum mechanics never departed from words and equations.[1] If anything, diagrams appear to be antithetical to what Dirac wanted to be "visible" in his thinking. Dirac was known for the austerity of his prose, his rigorous and fundamentally algebraic solution to every physical problem he approached. (Even his fellow physicists found his ascetic style sometimes to be too terse—in response to questions, he would repeat himself *verbatim*; other physicists sometimes complained that his papers lacked words.) Now it is not the case that diagrams are simply absent from physics. To cite one famous example, there is the famous diagrammatic-visual reasoning of theorists like James Clerk Maxwell who insisted that full understanding would only come when joined to imagined, visualizable machines running with gears, straps, pulleys, and handles. Maxwell wanted objects described and drawn that could, in the mind's eye,

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be grasped with the hands and pulled with the muscles. Similarly visual were Einstein's thought experiments, his use of hurtling trains, spinning disks, and accelerating elevators. Dirac's papers contain none of this. Not even schematic diagrams appear in his writings, visualizations of the sort that Richard Feynman introduced to facilitate calculation and impart intuition about colliding, scattering, splitting, and recombining particles.[2]

It would seem, then, that the corpus of Dirac's work would be the last place to look for pictures. But in the Dirac archives something remarkable emerges. I was astonished, for example, to find these comments penned by Dirac as he prepared a lecture in 1972: "There are basically two kinds of math[ematical] thinking, algebraic and geometric." This sounds like the theoretical twin of a contrast I have long pursued between laboratory methods that yielded images (analogous here to Dirac's geometric thinking) and those methods predicated on the logical or statistical compilations of data points (analogous to Dirac's algebraic thinking).[3] So I was intrigued. Given Dirac's austere public predilection for sparse prose, crystalline equations, and the complete absence of diagrams of any sort, I assumed that in the next sentences he would go on to class himself among the algebraists. On the contrary, he wrote in longhand,

> A good mathematician needs to be a master of both.
> But still he will have a preference for one rather or the other.
> I prefer the geometric method. Not mentioned in published work because it is not easy to print diagrams.
> With the algebraic method one deals with equ[ations] between algebraic quantities.
> Even tho I see the consistency and logical connections of the eq[uations], they do not mean very much to me.
> I prefer the relationships which I can visualize in geometric terms.
> Of course with complicated equations one may not be able to visualize the relationships e.g. it may need too many dimensions.
> But with the simpler relationships one can often get help in understanding them by geometric pictures.[4]

These pictures were not for pedagogical purposes: Dirac kept them hidden. They were not for popularization—even when speaking to the wider public, Dirac never used the diagrams to explain anything. Astonishing: across the great divide of visualization and formalism that has, for generations, split both physics and mathematics, we read here that Dirac published on one side and worked on the other.

The poverty of print technologies in and of itself seems rather insufficient as an explanation for the privacy of Dirac's diagrams, but in another (undated) account his characterization may be more apt: "The most exciting thing I learned [in mathematics in secondary school at Bristol] was projective geometry. This had a strange beauty and power which fascinated me." Projective geometry provided this Bristolean student new insight into Euclidean space and into special relativity. Dirac added, "I frequently used ideas of projective geometry in my research work in later life, but did not refer to them in my published work because I was doubtful whether the average physicist would know enough about them to appreciate them."[5] Lecturing in Varenna, also in the early 1970s, he recalled the "profound influence" that the power and beauty of projective geometry had on him. It gave results "apparently by magic; theorems in Euclidean geometry which you have been worrying about for a long time drop out by the simplest possible means" under its sway. Relativistic transformations of mathematical quantities suddenly became easy using this geometrical reformulation. "My research work was based in pictures—I needed to visualise things—and projective geometry was often most useful—e.g. in figuring out how a particular quantity transforms under Lorentz transf[ormation]. When I came to publish the results I suppressed the projective geometry as the results could be expressed more concisely in analytic form."[6]

So Dirac had one way of producing his physics in his private sphere (using geometry) and another of presenting the results to the wider community of physicists (using algebra). Nor is this a purely retrospective account. For there remains among his papers a thick folder of geometrical constructions documenting Dirac's extensive exploration of the way objects transform relativistically. These drawings are not dated but on their reverse sides are writings dated from 1922 forward. None of these drawings were ever published or, as far as I can tell, even shown to anyone (figs. 1 and 2).

The question arises: how ought we to think about Dirac's "suppressed" geometrical work? Dirac himself saw projective geometry as key to his entrance into a new field: "One wants very much to visualize the things which we are dealing with."[7] Should one therefore split scientific reasoning, as Hans Reichenbach did, between a "logic of discovery" and a "logic of justification"? For Reichenbach there were some patterns of reasoning that were, in and of themselves, sufficient for public demonstration. Other procedures, more capricious and idiosyncratic, could not count as demonstrations though they might serve the acquisition of new ideas.[8] This distinction saturates the philosophy of science of the postwar era. In Karl Popper's hands it helped to ground his demarcation criterion between science and non-science: only scientific theories, in the context of justification, were falsifiable, only in the realm of the justifiable was there anything dignified of the word *logic*. "My view," Popper wrote, "may be expressed by saying that every discovery contains 'an irrational element', or 'a creative intuition', in Bergson's sense."[9] By contrast, Gerald Holton took the private-scientific domain to have a sharply articulable structure that can be characterized by commitments to particular thematic pairs (such as continuum/discretum or waves/particles). According to Holton, this rich, three-dimensional space of private thought is then "projected" onto the plane of public science (defined by the restricted axes of the empirical and the logical). In this empirical-analytic public plane, much of the private dynamic of science is necessarily lost.[10] Recent work in science studies has either denied the force of the Reichenbachian distinction, or maintained the public/private distinction in other terms. For example, Bruno Latour, in his early work with Steve Woolgar, characterized

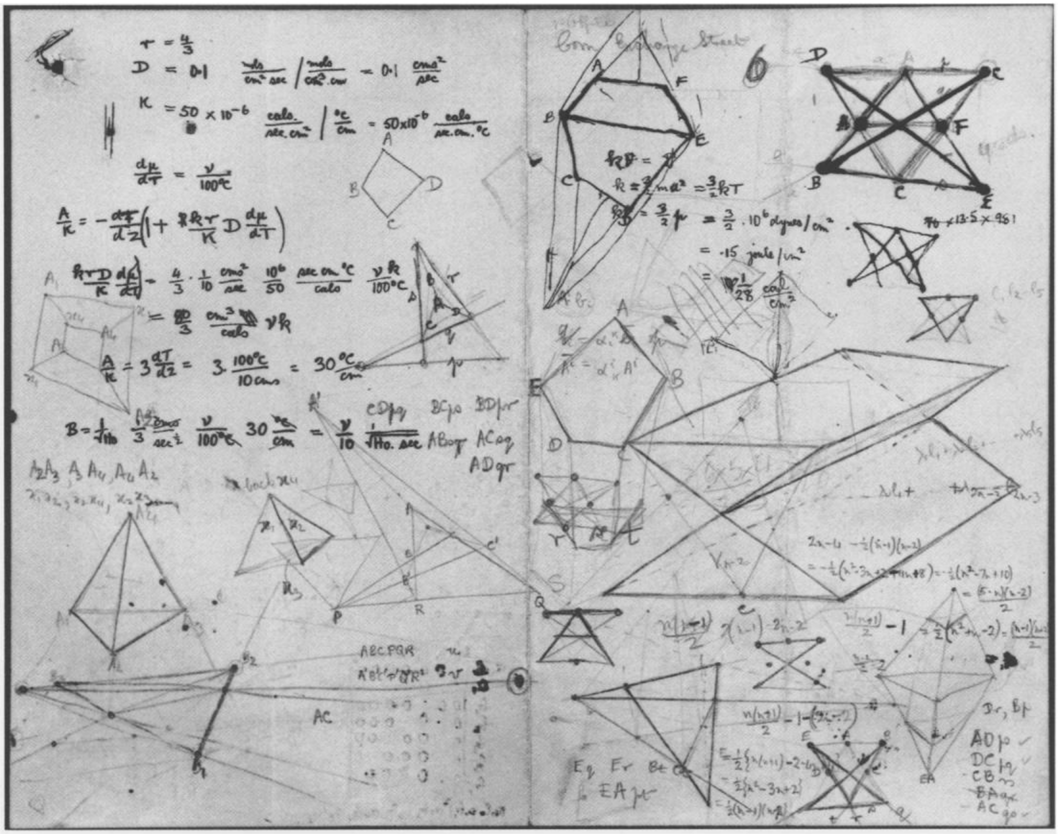

FIGURE 1. Paul Dirac, Geometrical Sketches, in the Paul A. M. Dirac Papers, Florida State University, Tallahassee, Florida; hereafter PDP. By permission of the Florida State University Libraries.

private science by a different grammar: the private is filled with modifiers, modal qualifications that slowly are filtered out until only a public, assertoric language remains.[11]

Certainly the common view of drawing as *preparation* would fit this sharp separation of public and private. Private sketches, in virtue of their schematic and exploratory form, would count as the precursors to the completed painting; private scientific visualization and sketches would, without requiring rigor, precede the public, published scientific paper. In such a picture the interior is psychological, aleatory, hermetic, and unrigorous while the exterior is fixed, formally constrained, communicable, and defensible. One thinks here of Sigmund Freud for whom the visual was primary, preceding and conditioning the development of language. To the extent that primitive reasoning is supplanted by language, the pictorial, uncon-

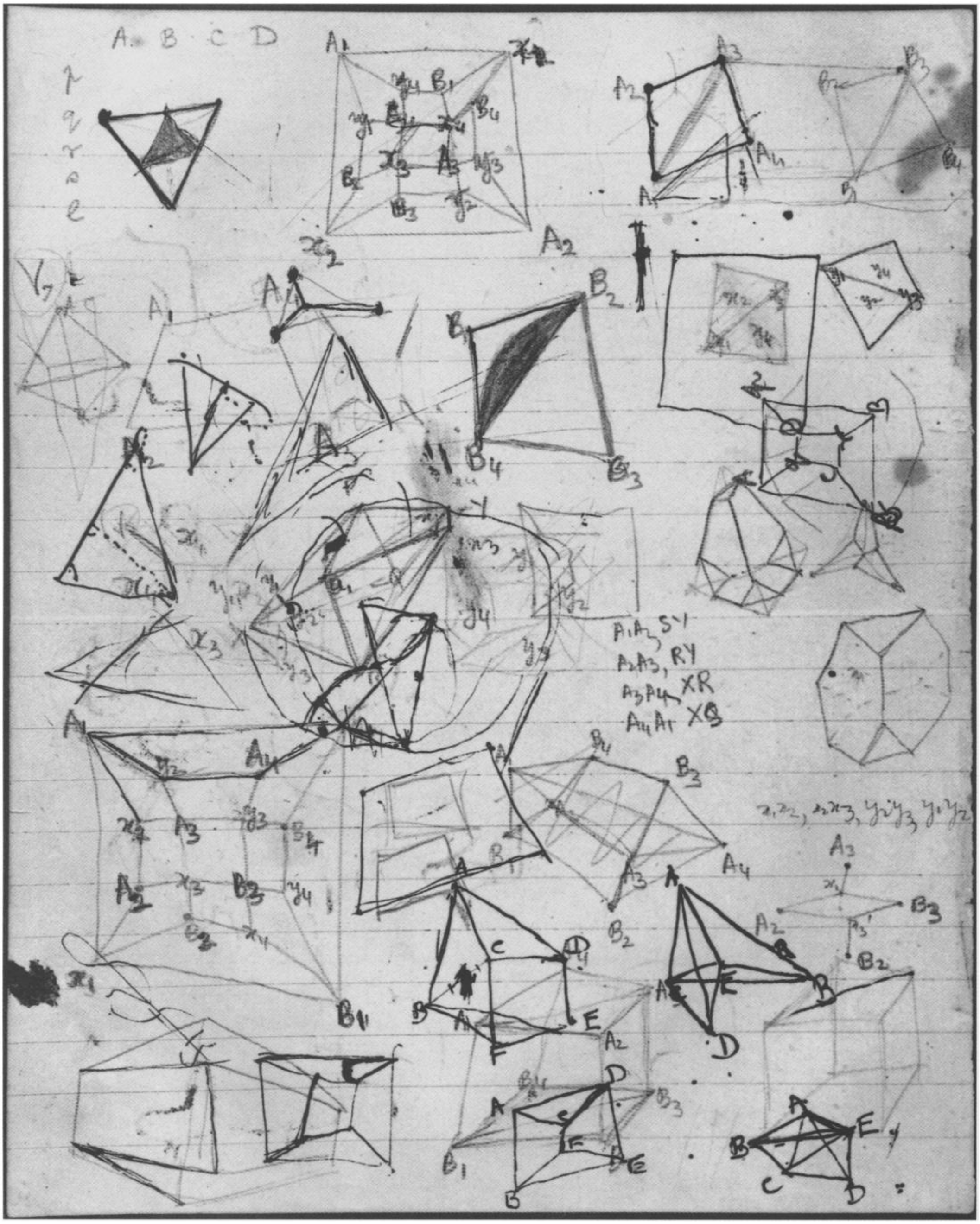

FIGURE 2. Dirac, Geometrical Sketches, in PDP. By permission of the Florida State University Libraries.

scious form of reason is of a different species from that of conscious, logical, language-based thought.

For some analysts of science, the advantages of the radical public/private distinction is that it brought the private into a psychological domain that opened it up to studies of creativity. For others, the separation permitted a more formal analysis of the context of justification through schemes of confirmation, falsification, or verification. For those who saw published science as merely the last step of private science, the distinction helped shift the balance of interest toward "science-in-the-making" and away from the published end product.

I want here to pose the question differently and, specifically, to challenge the search for intrinsic markers of scientific drawing that would make it in some instances "private" and in others "public." As we learn from Jacques de Caso's essay on Théophile Bra, Bra's drawings surely cannot be understood as the expression of a purely interior or subjective sensibility. For example, at least one of Bra's cosmological sketches was clearly tied to his views of public discussion about changes in the structure of Saturn's rings; Bra even wrote to the French astronomer and optician, Dominique-Francois-Jean Arago, about the problem.[12] Nor does the geometry of Dirac issue from an isolated form of reasoning. Dirac's fascination with projective geometry is anything but a private language in Ludwig Wittgenstein's sense—as we will see momentarily (fig. 3).

In both instances (Bra's cosmologies, Dirac's geometry) the drawings neither issue entirely from the public domain nor are they sourceless fountains from a reservoir of pure subjectivity. Tracking Bra's worldly iconological sources or Dirac's public sources in geometry would surely prove both possible and profitable. And *yet* there is something important in the circumstance that both Dirac and Bra constructed a domain of interiority around these practices. It is not that Dirac's geometric drawing or Bra's cosmogenic images were *intrinsically* interior or psychological—there is no separate logic here that could provide a universal demarcation criterion splitting the public from the private. Rather, both Dirac and Bra drew a line (so to speak) around their drawings. Both assiduously hid their pictures from the public gaze, and refused (in the case of Dirac) even to admit them into his published arguments. One suggestive concept helpful in capturing this delineation of the private might be Gilles Deleuze's notion of the fold. For Deleuze the "content" of what is infolded is not intrinsically separate from the exterior; there is no metaphysical otherness dividing inside from outside. Instead, interiority is itself the product of an outside pulled in, a process that Michel Foucault called subjectivation because it makes contingent, not inevitable, the formation of what is understood as self.[13]

I want to push this notion of infolding or subjectivation in two directions. First, my concern here is with an aspect of the private that bears on the epistemic, rather than one that posits lines of individuation that separate a self from others and the world. That is, what interests me is the historical production of a kind of reason

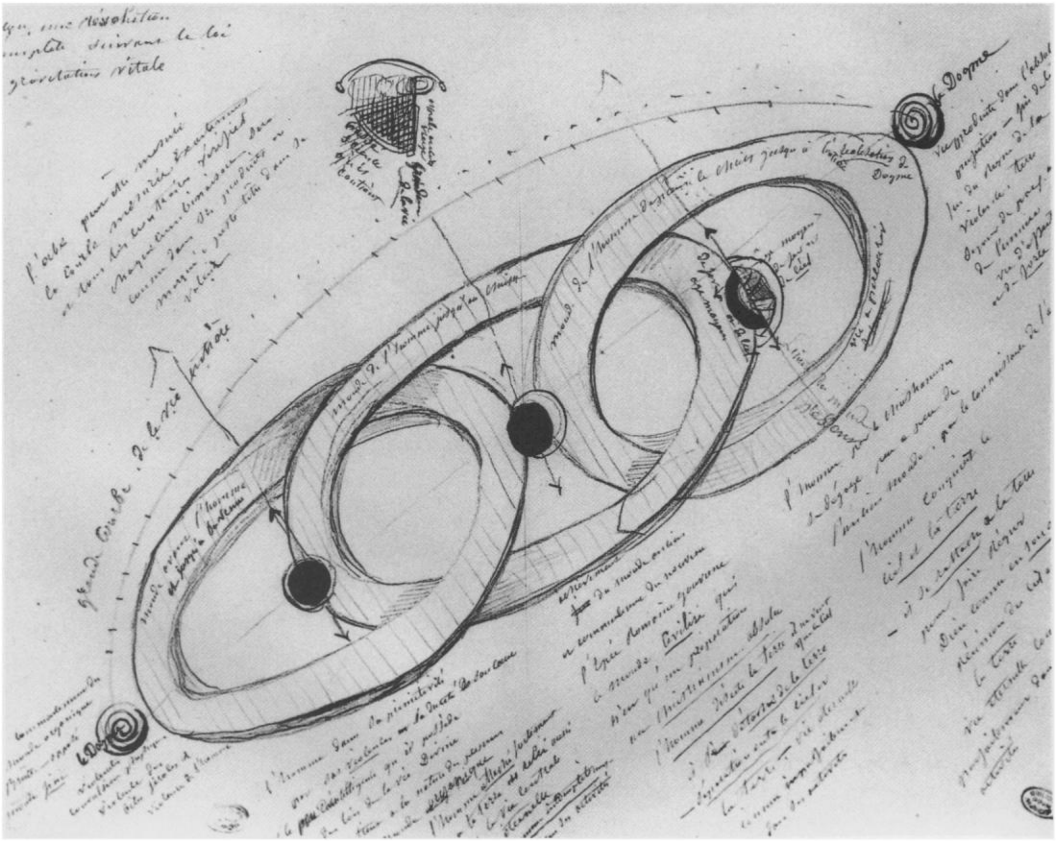

FIGURE 3. Théophile Bra, Untitled Drawing, in Jacques de Caso, *The Drawing Speaks: Théophile Bra, Works, 1826–1855* (Houston, 1997), plate 23.

that comes to count as private (rather than, for example, the production of the psychological sense of self more generally).[14] Second, building on this epistemic form of subjectivation, my concern is to explore the historical *process* by which this takes place. On such a view, the question shifts: How does a form of public inquiry and argument (geometry) come to count as private, cordoned-off reason?

Public Geometry, Private Geometry

The issue, therefore, is not what makes the interior or the private metaphysically distinct from the exterior and public, but rather how this inbound folding occurs over time. How, in our instance, did projective geometry pass from the status of a state religion at the time of the French Revolution to become, for Dirac, a repressed form of knowledge production that must remain consummately private—that is, how was geometry infolded to become, for Dirac, quintessentially an interior form of reasoning? What are the conditions of visibility that govern its place (or suppression) in demonstration?

So a new set of questions displaces those with which I began. Not the philosophical-psychological question: How do interior rules of combination differ from exterior rules of combination? But rather: What are the specific conditions that govern the separation of certain practices from the public domain? Not: How, linguistically or psychologically, does public science get created by successive transformations of the private domain? But rather the inverse: How do the "private" structures of visibility (specifically in drawing) get pulled in from the public arena to form a domain aimed, in the first instance, at the inward regulation of thought (rather than outward communication)? Consequently what we have is not quite the Deleuzian question either—not the transhistorical elucidation of what he calls the topology of the fold, but rather the historical process of the folding itself. What happens, over time and across places, such that features of public demonstration *become* private forms of reasoning?

During the late eighteenth century, descriptive geometry (later known as projective geometry) was first heralded by Gaspard Monge, as preeminent mathematician, as political revolutionary, and as director of the Parisian *Ecole polytechnique*. As Lorraine Daston and Ken Alder have shown, Monge's texts and the *Polytechnique* curriculum more generally were all oriented toward the school's mission to train engineers.[15] Descriptive geometry, the science of a mathematical characterization of three-dimensional objects in two-dimensional projections, was supposed to serve not only mathematicians and engineers but also the *Polytechniciens* who would become the nation's future high-level carpenters, stonecutters, architects, and military engineers.[16] For a generation of Monge's successors—Polytechnicien engineers including Charles Dupin, Michel Chasles, and Jean-Victor Poncelet—descriptive geometry became much more than a useful tool. Geometry, they contended, would hold together reason and the world.

For Monge and his school, physical processes including projection, section, duality, and deformation became means of discovery, proof, and generalization. This physicalized geometry defined a new role for the engineer as an intermediary lodged between the state and the artisan. Geometrical, technical drawing, "the geometry of the workshop" became at one and the same time a way of organizing the component parts of complex machines and a scheme for structuring a social and workplace order.[17] Geometry became a way of being as well as *the* proper way of founding a basis for mathematics. Indeed, at the *Ecole polytechnique*, geometry became an empirical science. Auguste Comte came to speak of an empirical mathematics, Lazare Carnot exploited physically motivated transformations in geometry and identified correlates between mathematical entities and their geometrical twins.

Geometry was practical and more than practical. Certainly for Dupin, Chasles, Poncelet, and their students, geometry towered above all other forms of knowledge as the paragon of well-grounded argumentation, *better* grounded, in particular, than algebra. Projective geometry came to stand at that particular place where engineering and reason crossed paths, and so provided a perfect site for pedagogy. As Monge insisted, projective geometry could play a central role in the "improvement" of the French working class—"Every Frenchman of sufficient intelligence" should learn it, and, more specifically, geometry would be of great value to "all workmen whose aim is to give bodies certain forms."[18] Enthusiastically Henri Saint-Simon and his followers adopted the cause in their utopian planning. Descriptive geometers established classes across Paris, joined the geometrical cause to republicanism, and launched a wider commitment to worker education. In 1825, Dupin proclaimed in his textbook that geometry "is to develop, in industrials of all classes, and even in simple workers, the most precious faculties of intelligence, comparison, memory, reflection, judgment, and imagination. . . . It is to render their conduct more moral while impressing upon their minds the habits of reason and order that are the surest foundations of public peace and general happiness."[19] Both before and after the French Revolution, geometry, as Alder notes, became the foundational skill in the training of workers—several thousand passed through the various popular art training programs. Geometry would teach both transferable skills crossing the trades and at the same time stabilize society by locking workers into the social roles previously occupied by fathers.[20]

Geometry did not, however, survive with the elevated status it had held in France at the highwater mark of the *Polytechniciens'* dominance. Analysts displaced the geometers. Among their successors was Pierre Laplace, for whom pictures were anathema and algebra was dogma. It was not in France, therefore, but rather in Britain and Germany that educators, scientists, and even politicians took up the cause of descriptive geometry with the conjoint promise of epistemic and pedagogical improvement. So although the French mathematical establishment had turned decisively to analysis in the last third of the nineteenth century, the British did not. Euclid had long reigned over British education as an exemplar of good sense and a pillar of mental training. By 1870, however, there was a widespread and disquieting sense that the British were losing to the Continent in the race for science-based industry. Geometry was no exception. In January 1871, leading mathematicians of the British Association for the Advancement of Science joined a committee known as The Association for the Improvement of Geometrical Teaching. Their goal was to produce a reform geometry better suited to technical and scientific education, in a form less rigid than that demanded by the purer mathematicians and enforced on schools. New methods of geometrical argument were introduced, and teachers began to step away from the definitions, forms of argument, and order of theorems dictated by the historical Euclidean texts. Such a loosening of Euclid's hold over the schoolchild's mind did not go undisputed. By 1901 the reformers (aiming to join geometry to the practical arts) and conservatives (hoping to preserve its purity) had settled into such powerfully opposed camps that separation seemed inevitable.[21]

These, then, were some of the nineteenth century's territories of geometry: Up until the 1860s or so, the French celebrated projective geometry as joining high reason with practical engagement of the working class; then this physicalized geometry faded from the scene. In Britain, accompanying the rapid expansion of industrial, technical education, Victorian descriptive geometry became the symbol and means of socio-educational uplift, improving the lot of young workers, including those of the working class. For the mathematician-logician Augustus De Morgan, for example, geometry was a route to knowledge in general—as he argued in 1868: "Geometry is intended, in education, . . . to [unmask] the tricks which reason plays on all but the cautious, *plus* the dangers arising out of caution itself."[22]

Over the last decades of the nineteenth century, the teaching of geometry in Britain gradually moved away from a rigid Euclid-based textual tradition toward a more expansive interpretation of geometry's basis. In part this shift issued from the marketplace. No longer would it be adequate for the teaching of geometry to exemplify sound reasoning as an end utterly unto itself. Instead, geometry came to have a practical significance as well—crucial for the upbringing of engineers, the upper tier of tradesmen, and scientists. One widely distributed encyclopedia of technical education put it bluntly: "It is impossible to overstate the importance of a knowledge of Geometry, forming as it does the basis of all mechanical and decorative arts, constituting, in fact, the grand highway from which the various branches of drawing diverge."[23] At the same time, part of the freeing of geometry from its purely descriptive roots was an increasing emphasis by reformers on "modern" methods including, prominently, non-Euclidean and projective geometry of higher dimensions. Pressured by both practical and research exigencies, geometry came to illustrate sound reasoning not by being purely descriptive of an ideal world, but rather by instantiating a reason best captured by a multiplicity of approaches.[24]

So much for the general historical condition of geometry as a very public epistemic ideal and educational method: as a defining feature first of republican and then working-class French pedagogy it continued into the 1870s and beyond in Germany, and re-emerged within the technical education movement of Victorian England. What, then, are the specific historical conditions under which drawing came to count for Dirac both as a reliable home of reason and as a "private" science, judged by him variously as too hard to print, too arcane for physicists to understand, insufficiently persuasive, or insufficiently concise to merit publication?

Dirac's trajectory in mathematical physics took him across several of geometry's territories, temporal-spatial regions where geometrical drawing was laid out differently from one to the next. The goal in following that arc is to see how it came to pass that what had been the most public of mathematical regimes could become, for Dirac as he moved across this shifting map of geometry's fortune, a most private refuge of thought. Here is an account that begins not with an assumed intrinsic dynamics of interior (psychological) style, but rather with the historical creation of a kind of science judged private: *the epistemic subjectivation of the geometrical.* This is, therefore, not so much an attempt to follow Dirac's biography, but rather to observe Dirac as a kind of movable marker in order to track the conditions under which reasoning through drawing came to be classed as something to be, in his word, "suppressed," interiorized, made to constitute the private scientific subject.

Zero in on Dirac as we turn from the generic Victorian British trade school to Dirac's secondary school, the Merchant Venturers' Technical College, in Bristol. This was where Dirac's father, Charles Dirac, taught, and where Dirac himself received his primary and secondary scientific-engineering education. Created out of various mergers of the Free Grammar and Writing School, the Merchant Venturers' Navigation School, and various forms of the Bristol Diocesan Trade and Mining School, Dirac's school had stabilized both its structure and name in 1894.[25] Charles Dirac took his degree at the University of Geneva and then, in 1896, came to Merchant Venturers' where he pursued a long career teaching French. A feared figure on the faculty ("a scourge and a terror" according to some of the students), Charles Dirac clearly reveled in the disciplined teaching of language—especially French, but others too, including Esperanto.[26] Dirac the younger often claimed that he simply stopped speaking to avoid having to perform at home in perfect, grammatically correct French. Dirac's wife put it this way: "His domineering father made it a rule to be spoken to only in French. Often he had to stay silent, because he was unable to express his needs in French. Having been forced to remain silent may have been the traumatic experience that made him a very silent man for life."[27]

Merchant Venturers', from its outset aimed, as such schools did across Britain, to provide a passage for students into specific trades including bricklaying, plasterwork, plumbing, metalwork, and shoemaking. Navigation had been central to its mission for decades, and continued to be of importance as did mathematics, chemistry, and physics.[28] In every way distant from British public education, this school was not, in mission, in curriculum, or in student body, designed to prepare the upper class for their stations in empire through a study of the classics. In the school archives of 1912, for example, there survives correspondence between Merchant Venturers' and the nascent University College, about the advisability of teaching firemen and preparing students for their Mine Manager's Certificates. "The more we do for the working classes," the then headmaster wrote, "the better for the university."[29] Like so many technical colleges around England, Merchant Venturers' held geometry front and center as a site for training in an appropriate, practical reason.

Paul Dirac entered Merchant Venturers' in 1914, at the age of 12, passing from it immediately into his study of electrical engineering at Bristol University, where the university's program was, in fact, run by Merchant Venturers' as an extension of their primary and secondary programs. Young Paul took up electrical engineering under the supervision of David Robertson; Dirac's notebooks show a diligent student, adept in the technical drawing that had accompanied geometry from France to Germany to England. Month after month, Dirac trained himself to confront the constant stream of practical problems: electrical motors, currents, shunts, circuits, generators. Graduating in June 1921, he had as his principal subjects electrical machinery, mathematics, strength of materials, and heat engines (fig. 4).[30]

While he was in the midst of this engineering program, Dirac watched Arthur Eddington's 1919 eclipse expedition, "hit the world with tremendous impact," and Dirac, along with his fellow engineering students, desperately immersed themselves in the new theory of relativity. They picked up what physics they could from Eddington; Dirac even took a relativity course with the philosopher Charlie D. Broad. The relativity Dirac seized upon was not that presented in Einstein's 1905 paper—it was not a relativity of neo-Machian arguments and *Gedankenexperimenten* about trains and clocks. No, what enthralled Dirac was Hermann Minkowski's space-time, relativity cast into the diagrams in which startling relativistic results issued from reasoning through well-defined, if not-quite Euclidean, geometry. The appeal of this geometrized relativity was no doubt doubled in virtue of the fact that Dirac himself had struggled, in vain, to formulate a consistent, physically meaningful four-dimensional space-time.[31]

While a student, Dirac did some practical engineering work with the British Thompson Houston Works in Rugby and on graduation applied there for a job for which he was rejected. But Robertson was impressed by young Dirac and, with his engineering colleagues at Merchant Venturers', tried to lure him further into their field. They were bested by the mathematicians, who offered to include Dirac, gratis, in their courses for two years.[32] Entranced by his Bristol mathematics instructor, Peter Fraser, Dirac seized on projective geometry as his favorite subject and immediately began applying it to relativity. More specifically, Dirac turned his attention to the geometrical version of relativity that Minkowski had developed and made so popular; with projective geometry Dirac could simplify the new space-time geometry even further.[33]

In 1923 Dirac moved out of Bristol and up to Cambridge, where as a physics research student at St. John's, he entered the research group of Ralph H. Fowler. Fowler immediately introduced Dirac to Bohr's theory of the atom. But it took no time at all for Dirac to gravitate, on the side, back to the geometry he had come to love at Bristol. At 4:15, once a week, aspiring geometers would join the afternoon geometry tea parties held by the acknowledged Cambridge master of the subject, Henry Frederick Baker. Baker himself had just authored the first volume of his multitome text on projective geometry where he announced that whatever algebra was included, the geometry was sufficient unto itself. It was a form of mathematics that, Baker judged, would naturally appeal to engineers and physicists.[34] Certainly this proved to be the case with Dirac; as Olivier Darrigol, Jagdish Mehra, and

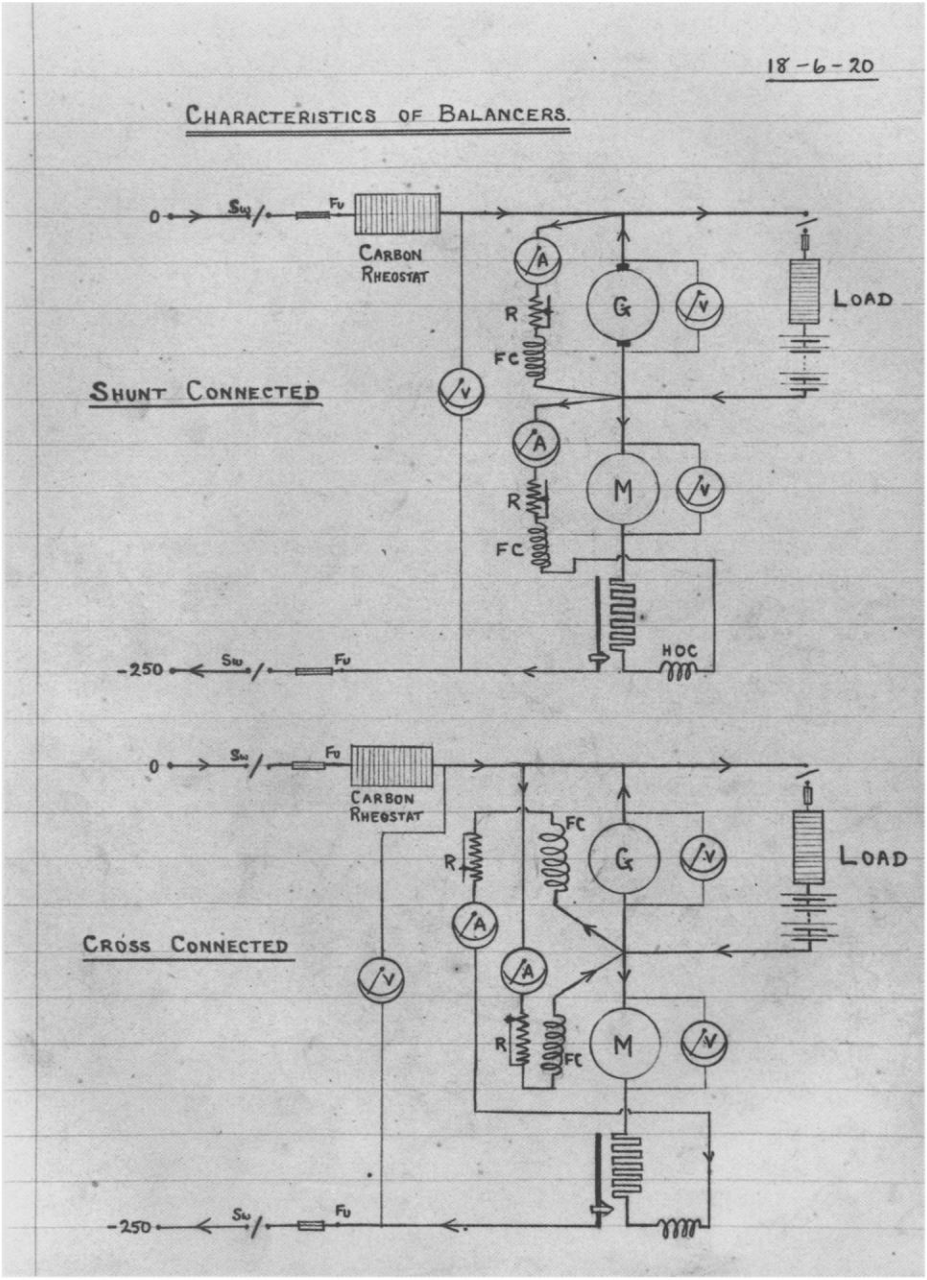

FIGURE 4. Paul Dirac, Electrical Engineering Diagrams. From Dirac's student notebooks, 18 June 1920, in PDP. By permission of the Florida State University Libraries.

Helmut Rechenberg have shown, even Dirac's notation seems to follow in some detail the choices made by Baker in his 1922 text.[35]

Sometime in 1924—the date cannot be deduced exactly from the handwritten fragment—Dirac delivered a talk to Baker's tea party. This was a tough audience to please. All of Baker's students and associates understood that silences would promptly be filled by grilling, and no quarter would be given in discussion.[36] Dirac immediately turned to the intersection of relativity with geometry and expressed his heartfelt sense that pure mathematics had nothing over the applied. On the contrary, so Dirac contended, there was a deep mathematical beauty in the specificity of the "actual world" that was obscure to the pure mathematician.[37] "I think," Dirac penciled onto his handwritten notes,

> the general opinion among pure mathematicians is that applied mathematics consists of finding solutions of certain differential equations which are the mathematical expression of the laws of nature. To the pure mathematician these equations appear arbitrary. He can write down many other equations which are equally interesting to him, but which do not happen to be laws of nature. The modern physicist does not regard the equations he has to deal with as being arbitrarily chosen by nature. There is a reason, {which he has to find} why the equations are what they are, of such a nature that, when it is found, the study of these equations will be more interesting than that of any of the others.

Old Newtonian gravity had a force that varied as the distance squared—but from the pure mathematician's view, there was nothing special about the square—it could have been cube or the fourth power. But the new theory of gravity, built out of Riemannian geometry, was (from the physicist's perspective) anything but arbitrary.[38]

"Again," Dirac added, "the geometrician at present is no more interested in a space of 4 dim[ensions] than space of any other number of dimensions. There must, however, be some fundamental reason why the actual universe is 4 dim[ensional], and I feel sure that when the reason is discovered 4 dimensional space will be of more interest to the geometrician than any other." Questions of applied mathematics, questions from the physical world, would, he believed, become of central concern to the mathematician. That which is arbitrary in pure terms became fixed, definite, and unique when put into the frame of a real-world geometry.[39] To draw diagrams, to picture relationships—these were the starting points for grasping why the universe was as it was.

These words would have been music to Baker's ears, for he had little truck with the new, vastly more abstract, rigorous, and algebraic mathematics that was coming into prominence. For example, when the Indian abstract number theorist Ramanujan wrote to the leading mathematicians at Cambridge, Baker had evinced no particular interest in him or his work. G. F. Hardy and J. E. Littlewood welcomed the unknown Indian number theorist as something of a mathematical prophet.[40] Hardy, who helped shape a generation of British mathematics, emphasized rigor, axiomatic presentations, and perfect clarity in definitions. By stark contrast, Baker began volume 5 of his famous series of works on geometry with the words, "The study of the fundamental notions of geometry is not itself geometry; this is more an Art than a Science, and requires the constant play of an agile imagination, and a delight in exploring the relations of geometrical figures; only so do the exact ideas find their value."[41]

Dirac's fascination with the confluence of physical reasoning, geometrical pictures, and mathematical aesthetics became a theme to which he returned throughout his life. In a fragment called "The Physicist and the Engineer," Dirac contended that mathematical beauty existed in the approximate reality of the engineer, not in the realm of pure and exact proof. Mathematical beauty was *the* guide but it was a guide through the approximate reality of the engineer's world, the one actual world in which we live. Many times Dirac insisted that *all* physical laws—Isaac Newton's, Einstein's, his own, were but approximations. "I think I owe a lot to my engineering training because it did teach me to tolerate approximations," Dirac recalled. "Previously to that I thought any kind of an approximation was really intolerable. . . . Then I got the idea that in the actual world all our equations are only approximate. . . . In spite of the equations' being approximate they can be beautiful."[42]

In a sense, Dirac's trajectory can be seen as a series of flights from world to world, flights away from home, no doubt from his dominating father specifically. Margit Dirac, his wife, recalled after Paul's death, that "The first letter he wrote to me [in 1935] after his father's death was to say, 'I feel much freer now.' "[43] But my interest is not in reducing Dirac's views to his familial relations, but rather in following Dirac's path as it traversed a series of worlds of learning, a path that left mechanisms for circuits, circuits for geometry, projective geometry for physics, and eventually projective geometry and engineering for an algebra-inflected physics. It was a path at once ever further from trade work and from home. Schematically, one might summarize Dirac's trajectory as taking him across a surface that folded the geometrical, drawn world of pictures into a private space beneath the algebraic structures of the new quantum physics:

Merchant Venturers' (technical drawing) →

Bristol Electrical Engineering (mechanical and circuit diagrams) →

Bristol Mathematics (projective geometry) →

Cambridge (relativity/projective geometry) →

Cambridge (algebraic structures of quantum mechanics).

It was in the final transition beginning in 1925, just a few months after his tea party talk, that Dirac interiorized and privatized geometry, making public presentation purely in the mode of algebra. From this moment on, Dirac spoke the public ascetic language in which he couched all of his great contributions to quantum mechanics. But he had no affective relation to algebra—it was, in his words, an equation language that for him "meant nothing." Reflecting back on the years since his Bristol days in projective geometry, Dirac told an interviewer: "All my work since then has been very much of a geometrical nature, rather than of an algebraic nature."[44] These are statements characterizing Dirac as a subject in mathematical physics, carving out what is simultaneously a language, an affective structure, a form of argumentation, and a means of exploring the unknown.

The final step toward abstraction and toward the algebraic world for which he came to be considered a heroic figure in physics began in 1925 when his thesis advisor, Fowler, received the proof sheets for a new article from young Werner Heisenberg. The crux was this: he had dispensed with the Bohr orbits, he had developed a consistent calculus of the spectra emitted by various atomic transitions, and he had extended Bohr's "old" quantum theory of 1913 to cover a vastly more general domain. For Dirac there was something else that had fascinated him in Heisenberg's paper—the mathematics. In the course of his calculations Heisenberg had noted that there were certain quantities for which A times B was not equal to B times A. Heisenberg was rather concerned by this peculiarity. Dirac seized on it as the key to the departure of quantum physics from the classical world. He believed that it was precisely in the modification of this mathematical feature that Heisenberg's achievement lay. It may well be, as Darrigol, Mehra, and Rechenberg have argued, that the very idea of a multiplication that depends on order came from Dirac's prior explorations in projective geometry.[45] Perhaps it was here that Dirac began to feel that he could recreate the public algebraic world in an interior geometrical one. In any case, from there Dirac was off and running with a new mathematics, accurate predictions, no (public) visualization at any level. On the side, geometry ruled.

Dirac's steps into the unvisualizable domain of quantum mechanics were taken with a certain ambivalence. As he generalized the basic equation of quantum mechanics to include relativity, as he accrued a sense of departing from safe land, the cost to him was movingly captured in an essay he wrote repeatedly over several years titled "Hopes and Fears in Theoretical Physics." In an early fragment Dirac scribbled:

> The effect of fears are perhaps not so obvious.
> The fears are of two kinds.
> The first one is the fear of putting forward a new idea which may turn out to be quite wrong.
> The fear of sticking one's neck out.
> perhaps having to retract and being exposed to humiliation.
> It may be that such a fear acts largely subconsciously
> and inhibits one from making a bold step forward.
> A man may get close to a great discovery and fail to make the last vital step.
> Possibly it is such a fear that blocks this step.[46]

In these highly inflected lines, Dirac explicitly touched on his own terror of the humiliating failure that abutted any chance of success, a terror expressed in an ambivalence at once drawn toward risk and success (in the form of the quantum theory he helped create) and yet recoiling with fear from possible failure and "sticking his neck" out from his own place of security. There is here a psychological story of the ambivalence of leaving home, a "home" that is conjointly familial, social, and epistemic—Merchant Venturers' was the workplace of his father, his training ground in engineering, and the place of his first encounter with the projective geometry to which Fraser (and later Baker) had introduced him.

But there is a further story that is only incompletely lodged in this geography of the psychological. This other narrative entails an account of how the logic of drawing was "suppressed"; how thinking through drawing diagrams went from being celebrated across Europe in the mid–nineteenth century to being marginalized at the beginning of the twentieth. To complete this broader narrative properly would take us into the shifting fortunes of geometry in France and Germany, and into fundamental changes in pedagogy at Cambridge.[47] I have only begun to sketch here the shifting role of persuasive visibilities in physics and their function in shaping an epistemological interior life for Dirac.

The Suppression of Geometry

To the mathematical generation that came of age after 1900 in England, geometry was no longer a science with claims to being descriptive of the world. Instead geometry, once the sun in the scientific sky, was being eclipsed by the formalized, devisualized system of logical relations exemplified on the Continent by mathematicians associated with David Hilbert and by physicists linked to Heisenberg. In Cambridge, it was Hardy who epitomized this new world of rigor—expressing the new mathematics in the formal relations of number theory, not in a descriptive, physicalized, and *drawn* geometry. By the early 1920s, drawn diagrams felt ever more like a disappearing trace, a vestige of a system of inquiry, pedagogy, and values that was fast fading from the Cambridge scene. For the historian of mathematics Herbert Mehrtens, the geometrical-intuitive mathematicians in many ways stood for a *Gegen-Moderne*, an antimodernism fighting to bind mathematics to the physical world and beyond—to psychology, pedagogy, and progressive technology. The moderns, he argues, wanted to bound and restrict mathematics, guarding their authority through a professional autonomy; mathematics, they argued, was not "about" anything exterior to its own formal structure.[48]

Dirac stood with one foot in the Cambridge of the older sort (through his association with Baker) and the other in the "new" Continent-leaning Cambridge (through his alliance with Heisenberg, Hilbert, and Hardy). It was a choice between Victorian geometrical tea parties and a post-Victorian modernism. Even as Dirac gave his own tea party talk in 1924, Baker's projective geometry was on the wane. Dirac had moved into the wing of Cambridge mathematics that had already lost the war to set the exam standards for the next generation of students and the mathematical standards for the next generation of researchers. Drawing diagrams gave Dirac an older safe point from which to venture into the new and, as he repeatedly emphasized, more fearsome unknown.

Heisenberg's paper of 1925 was antivisual without being, for that, formally and rigorously mathematical. It was physical and yet completely unvisual. Here was a final step away from the legacy of the *Ecole polytechnique*'s physicalized geometry, away from Felix Klein's tactile mathematical models that formed part of his Erlanger program, away from the British Victorian effort to make descriptive geometry into the centerpiece of skilled reason binding head and hand. And yet, as Dirac launched a long and extraordinarily successful career expressed entirely in the language of algebra, there was another Dirac, privately sketching, figuring, reasoning with diagrams, translating the results back into algebra, and all but burying the scaffolding around an interior furnished with formerly public effects.

My inclination, then, is to use the biographical-psychological story *not* as an end in itself, but rather as a registration of Dirac's arc from Bristol to Cambridge, to an identification with Bohr's and Heisenberg's Continental physics. In that trajectory, Dirac was sequentially immersed in a series of territories in which particular strategies of demonstration were valued. Bristol University was a step away from the technical drawing of Merchant Venturers', the whole electrical engineering curriculum with its codified, abstracted, applied physics removed drawing to a form of depiction less tied to quasi-mimetic technical renderings and linked instead to more functional, topological circuit diagrams. Bristol's applied mathematics again took Dirac further away from engineering, as did Heisenberg's matrix mechanics.

Technical drawings idealize by removing nonfunctional textures; circuit drawings drop any pretense of mimetic depiction—they are topological insofar as they represent relationships and use icons to refer to component parts. Actual spatial positions and distances do not matter. Projective geometry is also topological in this sense—the distances are eliminated from consideration and only intersections and their relative locations count. Projective geometry began in the domain of the physical, crept somewhat away in higher dimensions and its representation of non-Euclidean geometries. But Dirac kept bringing projective geometry back to the world, using it to track each new topic in mathematical physics across a long career.

When Dirac moved to Cambridge to begin studying physics, he took with him this projective geometry and used it to think. But that thinking had now to be conducted only on the inside of a subject newly self-conscious of its separation from the scientific world. Dirac's maturity was characterized again by flight, this time to Heisenberg's algebra, an antivisual calculus that at once broke with the visual tradition in physics and with the legacy of an older school of visualizable, intuition-grounded descriptive geometry. With an austere algebra and Heisenberg's quantum physics, Dirac stabilized his thought through *instability*: working through a now infolded projective geometry joined by carefully hidden passageways to the public sphere of symbols without pictures.

Freud often argued that what cannot be expressed in private is manifested in public. In a sense I am suggesting the contrary here: at the turn of the century in Britain, projective geometry was shifting away from the status of a state-endorsed liberal epistemology that joined university to factory and toward a form of knowledge that was distinctly second class. Physicalized geometry—geometry grounded in spatial intuitions, visualizations, diagrammatics—collapsed under the language of an autonomous science. In a sense Dirac's suppressed drawings were the hidden remnants of an infolded Victorian world. Public geometry became private reason.

Notes

I would like to thank Sharon Schwerzel, Beverly McNeil, and especially Joseph R. McElrath for their assistance with the Dirac Papers.

1. P. A. M. Dirac, *General Theory of Relativity* (Princeton, 1996); P. A. M. Dirac, *The Principles of Quantum Mechanics* (Oxford, 1930). For biographical and monographic studies on Dirac see, inter alia, Helga Kragh, *Dirac: A Scientific Biography* (Cambridge, 1990); Behram N. Kursunoglu and Eugene Wigner, *Reminiscences About a Great Physicist Paul Adrien Maurice Dirac* (Cambridge, 1987); Peter Goddard, ed., *Paul Dirac: The Man and His Work* (Cambridge, 1998); J. G. Taylor, ed., *Tributes to Paul Dirac* (Bristol, Eng., 1987); Olivier Darrigol, *From c-Numbers to q-Numbers: The Classical Analogy in the History of Quantum Theory* (Berkeley, 1992), esp. chaps. 11–13; Donald Franklin Moyer, "Origins of Dirac's Electron, 1925–1928," *American Journal of Physics* 49 (1981): 944–49, 1055–62, 1120–25; and Jagdish Mehra and Helmut Rechenberg, *The Historical Development of Quantum Theory*, 5 vols. (New York, 1982–87), esp. vol. 4.
2. On James Clerk Maxwell and mechanical analogies, see the following, along with references therein: P. M. Harman, *Energy, Force, and Matter: The Conceptual Development of Nineteenth-Century Physics* (Cambridge, 1982); C. W. F. Everitt, *James Clerk Maxwell: Physicist and Natural Philosopher* (New York, 1975); Crosbie Smith and M. Norton Wise, *Energy and Empire* (Princeton, 1989); Jordi Cat, "On Understanding: Maxwell on the Methods of Illustration and Scientific Method" (forthcoming). On Albert Einstein and visualization, see Gerald Holton, *Thematic Origins of Scientific Thought* (Cambridge, Mass., 1973); and on Richard Feynman, see S. S. Schweber, *QED and the Men Who Made It: Dyson, Feynman, Schwinger, and Tomanaga* (Princeton, 1994); P. Galison, "Feynman's War: Modelling Weapons, Modelling Nature," *Studies in the History and Philosophy of Modern Physics* 29B (1998): 391–434; and David Kaiser, Stick-Figure Realism: Conventions, Reification, and the Persistence of Feynman Diagrams, 1948–1964," *Representations* 70 (Spring 2000): 49–86. Schweber, *QED and the Men Who Made It*, chap. 8.
3. Peter Galison, *Image and Logic: A Material Culture of Microphysics* (Chicago, 1997).
4. Paul Dirac, "Use of Projective Geometry in Physical Theory," Boston, 30 October 1972, in Paul A. M. Dirac, Scientific Papers, Florida State University, Talahassee, Florida, file: Lectures B4 F26. Quoted with the permission of the Florida State University Libraries. Hereafter, the Dirac papers will be referred to as PDP.

5. Paul Dirac, Draft version "My Life as a Physicist," PDP, file: Lectures B3 F10.
6. Paul Dirac, "Recollections of an Exciting Era," three lectures given at Varenna, 5 August 1972. Quotations until "My research work" from typescript (10); and from that point forward from handwritten draft (3B). PDP, file: Lectures B4 F3.
7. Ibid.
8. H. Reichenbach, *Experience and Prediction* (Chicago, 1938).
9. Karl Popper, *Logic of Scientific Discovery* (New York, 1959), 32.
10. Holton, *Thematic Origins*, esp. chaps. 1, 3.
11. Bruno Latour and Steve Woolgar, *Laboratory Life: The Construction of Scientific Facts* (Cambridge, Mass., 1987). For challenges to the Reichenbachian distinction, see, e.g., Peter Galison, *How Experiments End* (Chicago, 1987), 3, 277; Karin Knorr-Cetina, *The Manufacture of Knowledge: An Essay in the Constructivist and Contextual Nature of Science* (Oxford, 1981), 84; Andrew Pickering, *Constructing Quarks: A Sociological History of Particle Physics* (Chicago, 1984), 414.
12. Jacques de Caso, "Process as Signification: The Drawings of Théophile Bra," in *The Drawing Speaks: Théophile Bra, Works, 1826–1855* (Houston, 1997), 26.
13. Gilles Deleuze, *Foucault* (Minneapolis, 1988), esp., "Foldings, or the Inside of Thought (Subjectivation)," 94–123.
14. For an excellent study of the historical attempt to foster individualism in nineteenth-century France, see the work of Jan Goldstein on Victor Cousin: Jan Goldstein, "Foucault and the Post-Revolutionary Self: The Uses of Cousinian Pedagogy in Nineteenth-Century France," in Jan Goldstein, ed., *Foucault and the Writing of History* (Cambridge, Mass., 1994); and Jan Goldstein, "Eclectic Subjectivity and the Impossibility of Female Beauty," in Caroline A. Jones and Peter Galison, eds., *Picturing Science, Producing Art* (New York, 1998).
15. Lorraine Daston, "Physicalist Tradition in Early Nineteenth-Century French Geometry," in *Studies in the History and Philosophy of Science* 17 (1986): 269–95; Ken Alder, *Engineering the Revolution: Arms and Enlightenment in France, 1763–1815* (Princeton, 1997), esp. 136–40; Bruno Belhoste, "Un modèle à l'épreuve, L'école polytechnique de 1794 au Second Empire," in Bruno Belhoste, Amy Dahan Dalmedico, and Antoine Picon, eds., *La formation polytechnicienne, 1794–1994* (Paris, 1994), esp. 12 ff.
16. Daston, "Physicalist Tradition," 279.
17. The "geometry of the workshop" is from Alder, *Engineering the Revolution*, 138.
18. Daston, "Physicalist Tradition," 291.
19. Ibid., 292.
20. Alder, *Engineering the Revolution*, 143–53.
21. John Perry, speaking to the British Association for the Advancement of Science in Glasgow in 1901, declaimed of the two sides: "What we want is a great Toleration Act which will allow us all to pursue our own ideals, taking each from the other what he can in the way of mental help. We do not want to interfere with the students of pure mathematics. . . . The more they hold themselves in their studies as a race of demigods apart the better it may be for the world. . . . I belong to a great body of men who apply the principles of mathematics in physical science and engineering; I belong to the very much greater body of men who may be called persons of average intelligence. In each of these capacities I need mental training and also mathematical knowledge"; cited in Joan Richards, *Mathematical Visions: The Pursuit of Geometry in Victorian England* (Boston, 1988), 197.
22. Cited in Richards, *Mathematical Visions*, 167.
23. *The Technical Educator: An Encyclopedia of Technical Education* (London, n.d.), 1:63.

24. Ibid., 175.
25. Information on flow charts, contained in letter from Mrs. P. M. Denney, Secretary to the Treasurer, The Society of Merchant Venturers, Merchant's Hall, to Mrs. R. Good, Librarian, Cotham Grammar School, 14 May 1999, and to author, 21 May 1999.
26. D. G. Pratten, *Tradition and Change: The Story of Cotham School* (Bristol, Eng., 1991), 24.
27. Margit Dirac, "Thinking of My Darling Paul," in Kursunoglu and Wigner, *Reminiscences*, 5.
28. Pratten, *Tradition and Change*, 11. Also: D.J. Eames, "The Contribution Made by the Society of Merchant Venturers to the Development of Education in Bristol," unpublished thesis, 1966, a copy of which is in papers of The Society of Merchant Venturers. My thanks to Mrs. P. M. Denney for making this available to me.
29. Julius Wertheimer to Sir Isambard Owen, 11 Sept. 1912, University of Bristol Special Collections. Happily we have a new interest in Victorian technical pedagogy—see, e.g., two excellent theses linking industrial arts with scientific-technical pedagogy: J. Graeme Gooday, "Precision Measurement and the Genesis of Physics Teaching Laboratories in Victorian Britain" (Ph.D. diss., University of Kent, 1989); and Nani Clow, "The Laboratory of Victorian Culture; Experimental Physics, Industry, and Pedagogy in the Liverpool Laboratory of Oliver Lodge, 1881–1900" (Ph.D. diss., Harvard University, 1999).
30. Faculty of Engineering, List of Candidates, 1910–1930, University of Bristol, Special Collections, DM 275.
31. Relativity, Dirac reported, felt like an escape from thoughts of World War I. It was in the immediate postwar period that he began to think that an extra (fourth) dimension might provide the link between space and time—though he had no sense that the metric was pseudo-Euclidean. Dirac in C. Wiener, ed., *History of Twentieth Century Physics* (New York, 1977), 109, cited in Abraham Pais, "Paul Dirac: Aspects of His Life and Work," in Goddard, *Paul Dirac*, 4.
32. R. H. Dalitz, "Another Side to Paul Dirac," in Kursunoglu and Wigner, *Reminiscences*, 71.
33. Peter Galison, "Minkowski's Space-Time: From Visual Thinking to the Absolute World," *Historical Studies in the Physical Sciences* 10 (1979): 85–121.
34. H. F. Baker, *The Principles of Geometry* (Cambridge, 1922).
35. See Mehra and Rechenberg, *Historical Development*, 161–68; and Darrigol, *c-Numbers*, 291 ff.
36. W. V. D. Hodge, "Henry Frederick Baker, 1866–1956," *Biographical Memoirs of Fellows of the Royal Society* 2 (1956): 49–68, on 53.
37. Paul Dirac, "Manuscript Notes for a Talk on Relativity to Prof. Baker's Mathematical Tea Party," draft lecture, c. 1924, Prof. Baker's Tea Party, PDP, file: Lectures B3 F25.
38. Ibid. Braces indicate cross-out in original.
39. Ibid.
40. Robert Kanigel, *The Man Who Knew Infinity: A Life of the Genius Ramanujan* (New York, 1991), 170.
41. Cited in Hodge, "Henry Frederick Baker," 56.
42. Paul Dirac, interview with Thomas S. Kuhn and Eugene Wigner, 6 May 1963, 5, in Archive for the History of Quantum Physics, Oral History Interviews, Niels Bohr Library, American Institute of Physics. Hereafter AHQP.
43. Dirac, "Thinking of My Darling Paul," 5.
44. P. A. M. Dirac, interview with Thomas Kuhn and Eugene Wigner, 1 April 1962, 3, in AHQP.

45. Darrigol, *c-Numbers*, 291–95.
46. Paul Dirac, draft lecture, 2 November 1971, "Hopes and Fears," PDP, file: Lectures B2 F19.
47. Andrew Warwick, "Cambridge Mathematics and Cavendish Physics: Cunningham, Campbell and Einstein's Relativity, 1905–1911," part 1: "The Uses of Theory," *Studies in History and Philosophy of Science* 23 (1992): 625–56, and part 2: "Comparing Traditions in Cambridge Physics," *Studies in History and Philosophy of Science* 24 (1993): 1–25.
48. Herbert Mehrtens, *Moderne Sprache Mathematik: Eine Geschichte des Streits um die Grundlagen der Disziplin und des Subjekts formaler Systeme* (Frankfurt am Main, 1990).