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Instantaneous electron energy distribution function in ion waves

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The assumptions involved in the use of Boltzmann's law to describe the electron density variations in ion acoustic waves are discussed. The need for electron collisions to fill the regions of maximum potential with low-energy electrons is pointed out. A direct method for measuring the electron energy distribution at any time during the period of an ion wave is presented. Experimental results show the existence of electrons with energy small enough for their motion to be constrained to the wave potential well. For higher energies, the distribution stays approximately the same. The method provides a direct measure of plasma potential, which is related to density. The Boltzmann law is still seen to be accurate for relative fluctuations of the order of up to 20%.

I. INTRODUCTION

In the ion sound wave problem[1–3] much attention has been paid to the ions, and experiments have concentrated on their fluctuations in density[4] and in the distribution function.[5] However, little is known about the electron behavior. Fluid theory[1] uses an electron pressure gradient term with a somewhat loosely determined compression coefficient γ. Linearized, collisionless kinetic theory[2] is not appropriate for electrons with kinetic energies smaller than the potential of the wave (that is, trapped electrons) when these particles, by moving with the positive potential well, contribute the largest part of the density fluctuation. A theory[6] accounting for the effect of a time-varying field has included trapped electrons. Since the electron collision frequency (with ions, neutrals, or walls) is often of the order of the wave frequency, these collisions can have a more important effect on trapping than a nonstationary field.

Thus, the description usually adopted[7] for electrons is the Boltzmann law:

$$n(x,t)=n_0\exp[e\phi(x,t)/KT], \qquad (1)$$

where $n(x,t)$ is the electron density, $\phi(x,t)$ is the wave potential, and T and $-e$ are the electron temperature and charge, respectively. For small values of $(e\phi/KT)$, linearization of (1) leads to

$$n_1/n_0=e\phi/KT, \qquad (2)$$

where $n_1=n-n_0$.

Relation (2) has been experimentally checked,[8] by using probes and relating floating potential to electron current.

Relation (1) has not been checked for larger values of ϕ. It involves a Maxwellian energy distribution. As we will briefly show, it is not clear whether or not this distribution should apply to any electron energy. This suggests looking at the distribution in the potential of an ion wave and, specifically, how it changes during the period. Such a measurement is the subject of this paper. It shows whether or not there is a temperature change, as suggested by the compression coefficient γ, if the energy distribution is conserved, and what the distribution for trapped electrons is. In addition to this information, it provides a direct measure of the plasma potential, which has not been done before.

II. A DISCUSSION ON THE ELECTRON ENERGY DISTRIBUTION IN ION WAVES

Let us first look at the validity of the Maxwell–Boltzmann distribution,

$$f(\mathbf{v},\phi)=n_0(m/2\pi KT)^{3/2}\exp[(e\phi-mv^2/2)/KT], \qquad (3)$$

where m is the electron mass.

It can be justified for ion waves in two limiting cases:

(i) The electron mean free path for electron–electron collisions is much shorter than the wavelength. Then, thermodynamic equilibrium applies. The electron–electron collision frequency ν_{ee} (Ref. 1, Sec. 5.3) can be expressed as

$$\nu_{ee}\simeq\omega_{pe}\Lambda^{-1}\ln\Lambda,$$

where Λ is the number of particles in the Debye sphere. Taking the mean electron velocity and choosing the velocity $(KT/m_i)^{1/2}$ for the ion wave and a frequency of the order of the ion plasma frequency ω_{pi}, the required inequality can be written as

$$\Lambda(\ln\Lambda)^{-1}\ll 1.$$

This condition is never fulfilled in the usual collective plasma phenomena.

(ii) In a purely collisionless plasma. Since we are interested in ion waves, ϕ is moving with a characteristic velocity much smaller than most of the electrons, that is, the field is almost stationary. One can thus write the Vlasov equation as

$$\mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{r}}+\frac{e}{m}\nabla\phi\cdot\frac{\partial f}{\partial \mathbf{v}}=0, \qquad (4)$$

where $\mathbf{r}$ is the position vector.

If the electric field is one dimensional (along x), the solution of (4) is a function of the energies,

$$f(\mathbf{r}, \mathbf{v}) = f(E_{\|}, E_{\perp}), \quad (5)$$

where

$$E_{\|} = \tfrac{1}{2} m v_x^2 - e\phi(x) \quad (6)$$

and

$$E_{\perp} = (m/2)(v_y^2 + v_z^2). \quad (7)$$

Then, let a reservoir of electrons be situated at infinity, where ϕ is zero and the distribution is Maxwellian with temperature T. Also let $\phi(x)$ always be zero or negative and have a single minimum. Then, owing to (5), the distribution function at any position x is given by Eq. (3).

However, to come closer to the wave situation, let us now assume that $\phi(x)$ has two negative, equal minima (or nulls) ϕ_m and one maximum ϕ_M in between. For a given electron, its energies $E_{\|}$ and $E_{\perp}$, Eqs. (6) and (7) are conserved. To get over the potential minima, and to reach the region between these, the energy of the electrons has to be

$$E_{\|} > -e\phi_m. \quad (8)$$

In this region, and in a position where the potential is ϕ, (8) implies, by Eq. (6)

$$|v_x| > v_m,$$

where

$$v_m = [(2e/m)(\phi - \phi_m)]^{1/2}. \quad (9)$$

That is, there are no electrons with velocities smaller than v_m, and the velocity distribution is a truncated Maxwellian

$$f(\mathbf{v}, \phi) = n_0(m/2\pi KT)^{3/2} \times \exp[e\phi - mv^2/2)/KT]\Gamma(|v_x| - v_m), \quad (10)$$

where $\Gamma(x)$ is the step function.

By integrating (10) over velocities, instead of Eq. (1), one obtains a new relation between n and ϕ:

$$n = n_0 \exp(e\phi/KT) \operatorname{erfc}[(e(\phi - \phi_m)/KT)^{1/2}], \quad (11)$$

where erfc is the complementary error function.

Paradoxically, relation (11) predicts a decrease in n with an increase in ϕ. If used instead of Eq. (1) in the coupled set of equations previously leading to ion waves, the solution is no longer progressive. Then, if ion waves are to exist, there must be trapped electrons with energies $E_{\|} < e\phi_0$, where ϕ_0 is the amplitude of the (oscillating) potential.

In order to fill the trap, collisions must be taken into account. If collisions with neutrals, ions, or walls are dominant, the following equation[9] is to be used:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}} + \frac{e}{m} \nabla\phi \cdot \frac{\partial f}{\partial \mathbf{v}} = \nu(f_{is} - f), \quad (12)$$

where ν is the collision frequency and f_{is} is the "isotropized" distribution function

$$f_{is}(v) = (4\pi v^2)^{-1} \int d^3v' \delta(|v'| - |v|) f(\mathbf{v}').$$

The collisional effect is expressed on the right-hand side of Eq. (12). It leads the distribution toward isotropy but not necessarily toward a Maxwellian distribution. Then, collisions can fill the trap by deplating the distribution for high energies.

III. MEASUREMENTS

The experiment is performed in a double discharge plasma[10] confined by the multipolar B field given by permanent magnets,[11] in a cylinder 22-cm diam and 70 cm in length. In the mid-plane two transparent plane grids 20-cm diam and closely spaced (1 mm), isolate the two halves of the column. The grid in contact with the plasma used as a source of ions, is floating. The grid in contact with the plasma where ion waves are propagated, is grounded. The plasma thus produced is homogeneous and B-field free in a column 10-cm diam, and the axial density gradient is reduced to $(n^{-1}dn/dx)^{-1} \simeq 50$ cm. The argon pressure is 3×10^{-4} Torr. The probe is a flat tantalum disk, 4-mm diam, whose plane is parallel and faces the grids, the back side being coated with a ceramic paste. When the Debye length is much shorter than the electrode dimension, and when the transit time in the sheath can be neglected, then the electron current I is a function of the applied negative potential V through

$$I = -es \int_{v_x}^{\infty} F(v_x') v_x' \, dv_x', \quad (13)$$

where s is the probe area, v_x' is the velocity component normal to the plane, and $F(v_x')$ is the one-dimensional velocity distribution,

$$F(v_x') = \iint dv_y \, dv_z f(\mathbf{v}),$$

$$v_x = [2e(\phi - V)/m]^{1/2},$$

where ϕ is the instantaneous and local plasma potential.

Using the parallel energy of an electron, Eq. (6), from Eq. (13) we get

$$\frac{dI}{dV} = \frac{eS}{m} F(E_{\|} = -\text{eV}). \quad (14)$$

Two types of information on the wave can thus be obtained from the probe characteristics:

(i) Its derivative with respect to V is a measure of the energy distribution.

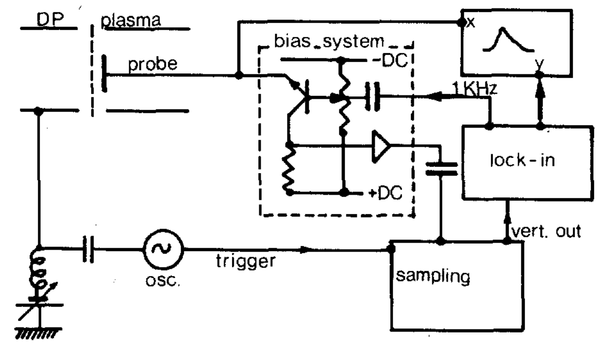

FIG. 1. Circuit diagram for the measurement of the instantaneous distribution function.

(ii) It provides a measure of the plasma potential. Indeed, Eq. (13) is defined as long as $\phi - V > 0$. When $V \geq \phi$, the probe collects all electrons, and the current $I(V)$ saturates. If there are electrons with velocities close to zero, the derivative dI/dV must decrease abruptly to zero when V rises above ϕ.

Experimentally, the abrupt change in dI/dV when $V = \phi$ is smoothed, mainly by the dispersion in work function on the surface of the probe. Assuming this dispersion to be random, and attributing the experimental smoothing to it, we get a mean square value of dispersion on the probe of $\Delta W \simeq 70$ mV. This value should be taken as the energy definition of the method.

The circuit diagram is shown in Fig. 1. An oscillator drives the ion wave with a frequency of 100 kHz, and sends a synchronous trigger signal to the sampling unit. The probe is biased through a transistorized circuit, the collected electron current being sampled at a definite time in the period of the wave. The probe potential is modulated with a low-frequency (1 kHz). The resulting current modulation is detected by the lock-in amplifier whose output, proportional to the sought-for derivative, is sent to the vertical axis of an XY recorder. The horizontal deviation is the negative of the probe potential.

A typical example of instantaneous energy distribution is shown in Fig. 2. Plasma density is 3×10^8 cm^{-3}, the electron temperature is 0.35 eV. The wave frequency is 125 kHz, the wavelength is 3 cm. To get an energy scale, horizontal deviation is (minus) probe potential and zero energy is arbitrary. The distance of the probe from the grids is 5.7 cm. The time in the period where distributions are measured, is monitored by biasing the probe at a positive potential, and looking at the instantaneous saturation current. This current is taken as proportional to the density.

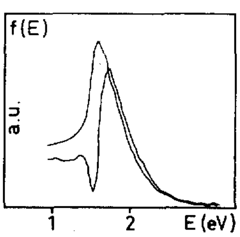

FIG. 2. Two energy distributions. Upper left curve is obtained at the time of maximum density; the other curve, at minimum density. The distribution (upper left) lying in between the two values of plasma potential, describes particles restrained in the potential well.

The upper-left distribution of Fig. 2, is obtained at the time of maximum density. Starting from the left (positive V), dI/dV has a constant level (nonzero since the "saturation" current is not independent of V). About an energy equal to plasma potential, it rises, reaches a maximum and then decays along an exponential. The other curve is plotted for the time of minimum density. Apart from a negative dip before plasma potential, it follows the same behavior. The total plasma potential variation $\Delta\phi$ in the period is measured here as being 140 mV.

Clearly, there are electrons having energies lying in between the potential extrema of the wave. Their motion along x is limited within a wavelength. However, they can leave the trap by transverse diffusion, the characteristic time for such diffusion being of the same order as the oscillation period in the potential well. Note also that the distribution for those electrons, as seen between the maxima of the two curves, remains on the same exponential: Maxwell's distribution still holds.

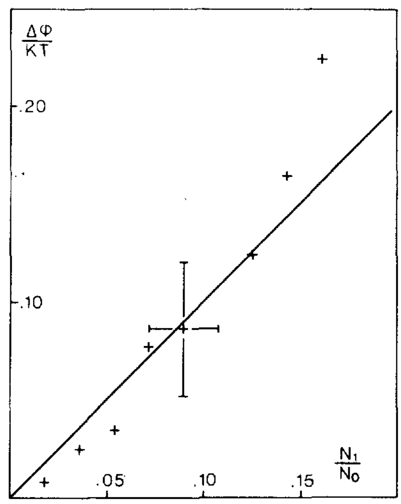

FIG. 3. Relative density and potential fluctuations. Circles are experimental points where $\Delta\varphi$ is obtained through measurements such as seen in Fig. 2, and N_1 from the electron saturation current on the probe. Solid line is the Boltzmann law, Eq. (16).

For electrons with higher energies, the two curves are close together, suggesting that f is only a function of $E_{||}$, as in Eq. (5). However, the distribution for higher density is slightly below the other. It is believed that such an effect could come from the filling of the trap by collisions, this filling would deplete the higher energies at the same time.

Since the energy distribution looks continuously Maxwellian for all energies, the Boltzmann law (1) should apply. It is approximately true in the case of Fig. 2, where the total density variation N_1, normalized to the equilibrium (wave excitation turned off) density

N_0 was measured to be 0.35, when $(e\Delta\phi/KT)$ is 0.4. This method of measuring $\Delta\phi$ has been used and related to electron density fluctuations in another experiment, where the plasma density was 5×10^8 cm^{-3}, the electron temperature was 2.9 eV, the wave frequency was 200 kHz and the wavelength was 1 cm. For each excitation amplitude, energy distributions as in Fig. 2 are plotted at times of maximum and minimum density. The displacement in energy of the $f(E)$ maxima, is taken as the measure $\Delta\phi$, and the variation $\tilde{I}$ of the saturation current I_s during the period is also recorded. The density fluctuation (N_1/N_0) is inferred from $(\tilde{I}/I_s)$. In doing this, attention is given to the fact that these two quantities are not equal. Indeed, since the slope of the saturation branch is not zero, $\tilde{I}$ is composed of two terms, one related to the actual density fluctuation, and the other to the plasma potential fluctuation. By allowing n and ϕ to be related through Eq. (1), the correction is

$$N_1/N_0=(\tilde{I}/I_s)[1-(KT/eI_s)(dI_s/dV)]^{-1}. \quad (15)$$

By changing the excitation amplitude of the ion wave, $\Delta\phi$ was increased from 25 to 650 mV. The measured values of $(e\Delta\phi/KT)$ and N_1/N_0 are plotted in Fig. 3. If the experimental points are to be compared with the Boltzmann law, Eq. (1) has to be written for the total fluctuation of density N_1, related to the mean value N_0, i.e.,

$$N_1/N_0=2\tanh(e\Delta\phi/KT). \quad (16)$$

Equation (16) is shown in Fig. 3 as the solid line. Thus, the measured quantities are seen to agree with Eq. (16) within experimental error.

IV. CONCLUSIONS

We have demonstrated the existence of electrons with energies low enough to be trapped in an ion wave. It is believed that their appearance is due to collisions. Within the limits set by the definition of the work function on the probe surface, their distribution looks Maxwellian. The distribution for higher energies is also Maxwellian. It is not exactly conserved during the period. However, there is no significant change in the temperature, and as far as density and potential are concerned, the Boltzmann equation (1) still gives an accurate description. As to the filling process of the trap, while theory has considered the effect of a time varying field,[6] the experimental situation suggests looking at isotropizing collisions through a model such as that given in Eq. (12).

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