

Material Models of Immaterial Things

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There is a dog-bites-person story about models that we all know very well. Faced with a complex physical system, we invoke a stripped-down system of abstractions to explain it. We recognize that Earth is not spherically symmetric. Yet assuming that it is composed of a series of concentric shells offers a tremendous simplification, one good enough for many purposes. We know that crystals are not geometric but reason successfully with cubic structures; we recognize that fluids in many circumstances are somewhat compressible but build descriptions of them with incompressible fluids to account for many phenomena.

This form of representational abstraction is at the center of so much of the continuing revolution, set in motion by Descartes, Galileo, and Newton, that it has become second nature to us—invoking abstractions of these ideal types to get at our messy, worldly reality is in many cases just what we mean by explanation.¹ Models like these—idealizations—seem to be at the heart of this enterprise of explanation.²

Once in a while, far less often than such “normal” cases, we encounter a form of modeling that seems, on the face of it, to be utterly backward: the modeling of an abstract, immaterial entity by a material system. Why would one use a physical entity, a real-world concatenation of gears and springs, of flowing water in rubber tubes, of straps, springs, and pulleys, to model something that itself was supposed to be captured by differential equations? At first sight, it might seem that a material model is “merely” an aid to the feeling of having grasped a purely theoretical account. Indeed, there is a philosophical-scientific tradition that takes understanding itself to be nothing more than a pedagogical, popularizing, or psychological crutch.³

To see the value of material models, we need a shift of priorities: from the near-ubiquitous hunt for explanation to the less-attended-to epistemic virtue of understanding. Years ago, Michael Friedman took a shot at adjudicating the relationship of understanding to explanation, depicting understanding as important—not just a decorative complement. But he kept understanding tied to explanation. Indeed, he

saw in the literature three ways of making the link from explanation to understanding. One way of specifying understanding is to say that understanding is present when a phenomenon can be deduced from another more securely grasped range of phenomena. A second passage from explanation to understanding takes place when we can show that some novel phenomenon can be tied back to a familiar one (even if not deductively), for example, we explain thermodynamic laws in terms of the collision of miniature billiard balls. Yet a third school says that understanding is present when it relates new phenomena to some historically specific (changing) set of prevalent conceptions. Friedman's own view is that all three notions of understanding fail because all involve a *local* explanation of something new in terms of something accepted. By contrast, Friedman's own position was that we understand something when we *unify* it vertically into the comprehensive, global set of scientific conceptions such that "our over-all understanding of the world is increased; our total picture of nature is simplified via a reduction in the number of independent phenomena."⁴ The view that explanations must be present to have understanding—or even that understanding is to be identified with explanation—remains widespread.

A more promising approach—as I see it—is taken by Henk de Regt, who values understanding as an epistemic goal in its own right. He identifies understanding (applied to theories) as closely allied with *intelligibility*: the features of a theory that are taken in a historical period to facilitate the theory's use.⁵ This is in line with Jordi Cat's approach to James Clerk Maxwell's use of metaphor and illustration: they help establish intelligibility, and with that intelligibility comes a precondition for explanation. "Understanding then establishes the cognitive basis for explanatory value."⁶

Like de Regt, Cat, and a growing number of others, I want to focus on understanding—though even less than they do on explanation and more on physical modeling and its links with other approaches to knowledge. Understanding, I will argue here, lies in this *coordination without homogenization*.⁷ As one can imagine, the etymology of *understand* is much discussed and disputed, but one strand, the one I find most intriguing, carries with it the vestige of literally "standing among." This is preserved not only in the German *verstehen* (stand among), both and in other languages, too, deriving from Proto-Indo-European **nter*, designating "between" or "among" (not just literally "beneath").⁸ But etymologically bolstered or not, it is this sense of the concept that seems most central to me: to understand is to stand among—and eventually to advance among—forms of knowing, drawing together the symbolic, the visual, and the material.

From quantum field theory to the British economy, from the nineteenth-century all-pervasive electromagnetic ether to contemporary accounts of black holes, I

would like to explore some of these surprising moments of modeling the abstract by the concrete.⁹ The goal is to grasp this back-and-forth between material models and immaterial things, the lateral correspondence that makes up at least this one, often peculiar part of what we mean by understanding.¹⁰

SYMBOL, SPACE, AND BODY

In his “Dynamical Theory of the Electromagnetic Field” (1864), James Clerk Maxwell turned from the examination of his twenty equations (of electrical and magnetic force, resistance, continuity, and more) to phenomena solidly within the domain of gears and pulleys. In the past, he had spoken of real strains and motions in the ether. Here he wanted to be more abstemious: “I wish merely to direct the mind of the reader to mechanical phenomena which will assist him in *understanding* the electrical ones.” All such invocations were, he insisted, “illustrative” and not “explanatory.”¹¹

In September 1870, in his Liverpool Presidential Address to the British Association, Maxwell reflected on the different kinds of people who reason about the physical world. True, the scientist is willing to act like a “calculating machine” for a time. Such procedural manipulations can make things clearer. But the act of one justified step after another can leave one without a synoptic vision of the whole; there are times when the steps are so numerous that a person “is sure to forget before he has reached the conclusion.” That is the moment to put machine-like reasoning aside and turn elsewhere: “to *understand* the subject by means of well-chosen illustrations derived from subjects with which he is more familiar.” Such “scientific illustrations” are, the Victorian polymath contended, invaluable. They “enable the mind to grasp some conception or law in one branch of science, by placing before it a conception or a law in a different branch of science, and directing the mind to lay hold of that mathematical form which is common to the corresponding ideas in the two sciences, leaving out of account for the present the difference between the physical nature of the real phenomena.” In this way, one could attain knowledge “more profound than could be obtained by studying each system separately.”¹²

True, Maxwell noted, there were those who, faced with a mathematical relation or law, no matter how complex, could grasp, purely from the symbolic, abstract representation alone, the full meaning carried by formal symbols. “Such men,” Maxwell noted, “sometimes treat with indifference the further statement that quantities actually exist in nature which fulfil this relation. The mental image of the concrete reality seems rather to disturb than to assist their contemplations.” Most of humanity, however, could not, absent a particular training, seize these relations, much less “retain in their minds the unembodied symbols of the pure

Equations of Electromotive Force.

$$\left. \begin{aligned} P &= \mu \left(\gamma \frac{dy}{dt} - \beta \frac{dz}{dt} \right) - \frac{dF}{dt} - \frac{d\psi}{dx} \\ Q &= \mu \left(\alpha \frac{dz}{dt} - \gamma \frac{dx}{dt} \right) - \frac{dG}{dt} - \frac{d\psi}{dy} \\ R &= \mu \left(\beta \frac{dx}{dt} - \alpha \frac{dy}{dt} \right) - \frac{dH}{dt} - \frac{d\psi}{dz} \end{aligned} \right\} \dots\dots\dots (D).$$

FIGURE 2.1. Maxwell's equations of electromotive force, from "A Dynamical Theory of the Electromagnetic Field, 1864," in *The Scientific Papers of James Clerk Maxwell*, ed. W. D. Niven (1890; repr., New York: Dover, 1965), 1:559.

mathematician." Indeed, if science was to "become popular, and yet remain scientific, it must be by a profound study and a copious application of those principles of the mathematical classification of quantities which, as we have seen, lie at the root of every truly scientific illustration." For Maxwell, to render physics "popular" did not imply merely bringing these relations to the masses. Instead, making the discipline popular meant altering the science for physicists who were not, could not, be satisfied with differential equations alone.

Of course, Maxwell allowed that "there are . . . some minds which can go on contemplating with satisfaction pure quantities presented to the eye by symbols, and to the mind in a form which none but mathematicians can conceive."¹³ Maxwell believed that the pure algebraic-mathematical mind found the physical instantiation of an equation to interfere with their "contemplation." No better example comes to mind of these complex abstractions than the equations that now bear Maxwell's name—among them his expression for the electromotive force for a moving conductor (Figure 2.1).

But there were other ways to grasp the physical situation that did not depend exclusively on differential equations. This second kind of mathematical mind, also thrived in abstraction, but in a way that was not at all algebraic. "Others," he continued, "feel enjoyment in following geometrical forms, which they draw on paper, or build up in the empty space before them."¹⁴ Maxwell drew such forms to illustrate how the lines of force could be derived from the contours of constant values of the potentials (Figure 2.2).

Third, and in Maxwell's categorization, finally, were those among the scientists who had a different conception altogether, one that Maxwell described rhapsodically:

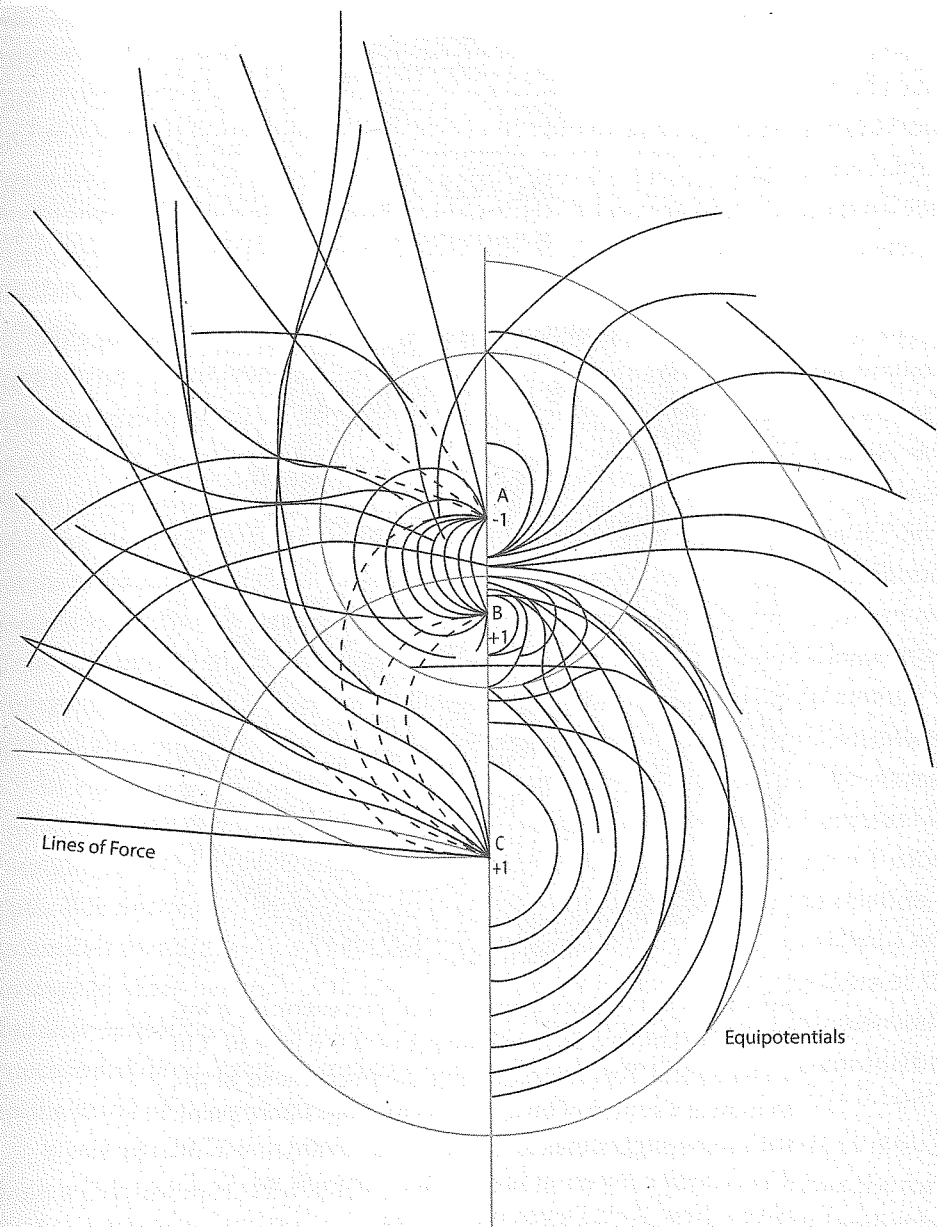


FIGURE 2.2. Maxwell's geometry of force. Two spheres with opposite charges: A, impermeable, B, a perfect conductor. Forces are orthogonal to the contours of equal potential.

"Others . . . are not content unless they can project their whole physical energies into the scene. . . . They learn at what a rate the planets rush through space, and they experience a delightful feeling of exhilaration. They calculate the forces with which the heavenly bodies pull at one another, and they feel their own muscles straining with the effort." Maxwell wrote of those who wanted this feeling of

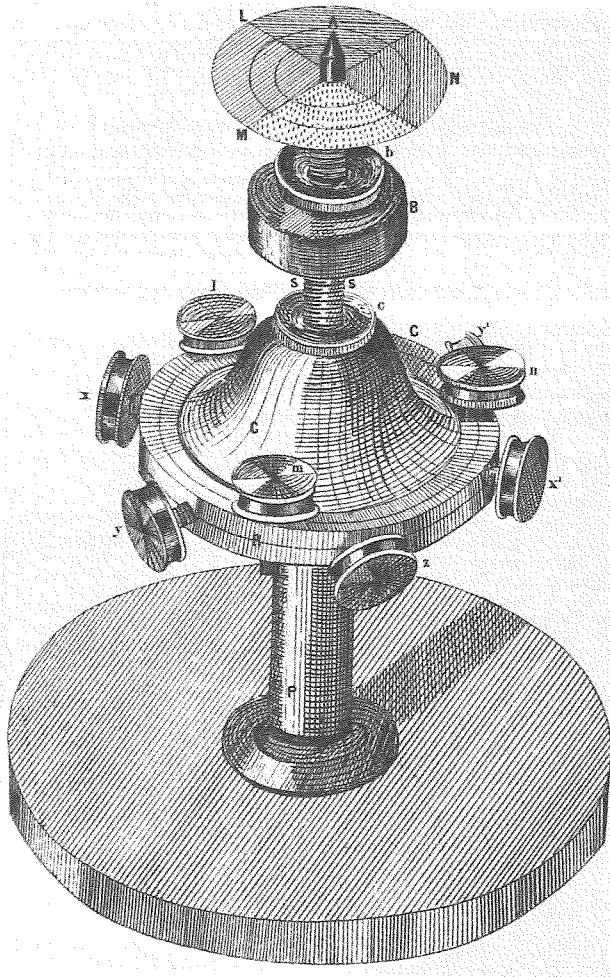


FIGURE 2.3. Poinot's astronomical precession in a toy. Maxwell exhibited an instrument/toy of this type in "On a Dynamical Top, for exhibiting the phenomena of the motion of a system of invariable form about a fixed point, with some suggestions as to the Earth's motion, 1857," in *The Scientific Papers of James Clerk Maxwell*, ed. W. D. Niven (New York: Dover, 1965), 1:262.

empathetic bodily response: "To such men momentum, energy, mass are not mere abstract expressions. . . . They are words of power, which stir their souls like the memories of childhood."¹⁵

One thinks here of Maxwell's own work echoing in many of his studies. In April 1857, he turned to the dynamical top, returning again to the image of a childhood contemplation of matter in motion. "The mathematicians of the last age, searching through nature for problems worthy of their analysis, found in this toy of their

youth, ample occupation for their highest mathematical powers.” Indeed, as he insisted, “no illustration of astronomical precession can be devised more perfect than that presented by a properly balanced top.” Here he credited French physicist and mathematician Louis Poinsoot (who had shown how forces on a rigid body could be captured by a single force and couple) for having brought to the subject the “power of a more searching analysis” than anything mere calculus could provide (Figure 2.3). “Ideas take the place of symbols, and intelligible propositions supersede equations.”¹⁶

This toy-instrument—a form of illustration in Maxwell’s sense—was key to understanding and a source of pleasure. Sigmund Freud once said that the original, true, and greatest happiness was the fulfillment of a childhood desire, the reengagement of that desire fulfilled in focused, serious, committed play. Of course, says Freud, the form this play takes is ever-shifting with circumstances: the pleasure of concentrated play shifts over time, but it always has a “date-mark” on it.¹⁷ So it seems to be for Maxwell, as he invoked the pleasurable “memories of childhood” associated with bodily expression drawn up into concepts. Maxwell’s *Traum* not *Trauma* found its site in the attachment of abstractions—wound up in momentum, energy, mass—to real kinesthetic, muscular, *physicalized* action, often coming to ground, so to speak, in the manipulation of toys—become—physicalized illustrations.

In April 1879, when Maxwell examined the electrical potential, he wanted something one could feel—the way one could (so he contends) feel pressure by plunging into the depths of the sea in a diving bell or feel temperature in a Turkish bath. But electrical force seemed mired in abstraction, referring to the relation of quantities measured at two different points; he worked insistently to afford a way to engage kinesthetically with electrical force. “It is *modelled* on the lines of the familiar definition of mechanical force, and those who find that they understand mechanical force better by ‘feeling what it is like’ can easily apply the same method to the study of electromotive force.”¹⁸

It was just this feeling “what it is like” that Maxwell was after when, in 1858, he built a physical model designed to illustrate why the rings of Saturn had the form and stability they did (Figure 2.4). Again, physical modeling by *felt* mechanical force lay central to the struggle to understand. “Illustrations,” for Maxwell, therefore had a technical specificity: they were not just the kind of one-off metaphors that swarm popular science. Instead, as Maxwell put it in 1870, “the correctness of such an illustration depends on whether the two systems of ideas which are compared together are really analogous in form, or whether, in other words, the corresponding physical quantities really belong to the same mathematical class.”¹⁹ For Maxwell, this congruence, under a common sheltering mathematical structure, was what led to a deeper understanding than any particular physical realization could provide.

This class relation, this relation of different realizations, is what I have in mind with the idea of a *lateral coordination* of approaches to a phenomenon—each differently articulates that which is investigated, for example, the rings of Saturn or the relation of magnetism to electricity. Together, the various modes deepen understanding by multiplying the ways of carrying on (rather than any form of vertical explanation by way of unification or reduction).

Or put another way, having different realizations of knowledge (algebraic, geometric, physical) with a mathematical commonality deepens understanding. The “more profound” knowledge might include but is not necessarily a matter of prediction or explanation. It is something else because it can be *generative*, productive of new forms of scientific work—not just the quantitative anticipation of the behavior of a physical system, nor even the particular explanation of why something occurred. A computer simulation could well produce a very precise quantitative prediction without, for that, creating new domains of application. Indeed, it is this eruption of the new out of the joining of two or more systems that signals the real power of understanding or, for our purposes here, modeling. Sometimes these various modes of thought combine in one person. Maxwell, after all, reasoned by turn geometrically, formally, physically. But, as he suggests, the power of this multimodel understanding can occur in the community, the assembly of scientists who know differently, focusing, like a lens collecting light, their various styles of knowing.

GENERATIVITY

Maxwell himself rejected the idea that the “mind of man” is usefully likened to a computation. The mind (for Maxwell) is not similar to Fourier’s heated body, “settling down into an ultimate state of quiet uniformity” that with the right laws one could have anticipated. No, Maxwell insisted, the scientific mind is more like a tree adapting to circumstances, “shooting out branches” toward the sky, plunging roots down into the soil. It is only given to us to “breathe . . . the spirit of our own age,” to seize “the characteristics” of our “contemporary thought.” Just to be prepared for these *un-*anticipated developments, it is crucial that mathematics and physics sort out which ideas from one domain of physics can be deployed in another.²⁰

Maxwell called the “figure of speech” of thought and language that effects this correspondence among various “departments” of science “scientific metaphor” in just those cases where each term of the metaphorical structure keeps the formal relations across departments. When that formal correspondence works, the method is “truly scientific,” by which Maxwell means “capable of generating science in its turn.”²¹

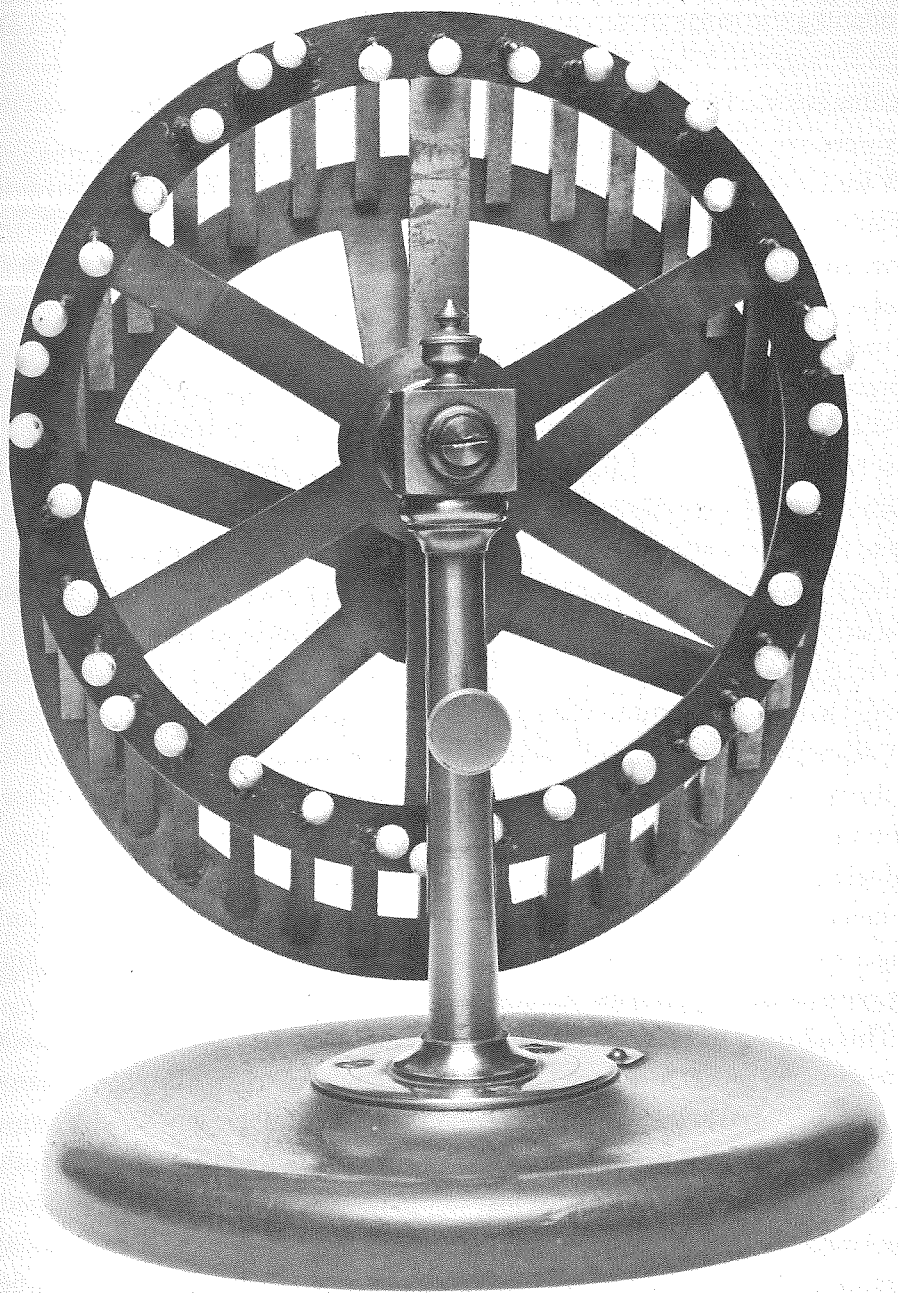


FIGURE 2.4. Maxwell's 1858 model of the movements of satellites that make up the rings of Saturn, from "Letter to William Thomson, 30 January 1858." Courtesy of and copyright the Cavendish Laboratory, University of Cambridge.

Bruce Hunt nicely captured Maxwell's ambition for models—as a means of broadening the range of acceptable paths forward: through instruction but also original research. When Maxwell vaunted illustrative models as “convenient for teaching science in a pleasant and easy manner,” one might take this to mean that they were *only* of pedagogical interest. That is, one might (Maxwell does not) think they were effective in *transmitting* knowledge but not think of them as having epistemic virtue in and of themselves: valuable for gaining, securing, and further advancing knowledge. Such a reduction to teaching or popularizing is clearly *not* what he meant. Indeed, for Maxwell, the illustrative model offered a path for moving mathematical structures from one site to another—around a common physical interpretation—without sacrificing force of argument: “If science is ever to become popular, and yet to remain scientific,” there had to be illustrations of this type. It must be by the use of physical illustrations, for these provided the only means by which many areas of physics could be presented in a rigorous yet understandable way to those for whom a bare set of equations would be wholly opaque.²²

For the Maxwellian Oliver Lodge, like Maxwell, it was the overarching mathematical regularities that bound the flow of material liquids to the flow of immaterial electricity. Electricity, he asserted, “behaves like a perfect and all-permeating liquid. Understand I by no means assert that electricity is such a fluid or liquid; I only assert the undoubted fact that it behaves like one, i.e. it obeys the same laws.” Of course (says Lodge), we must be on the lookout for discrepancies, but absent some variation, we ought to pursue the joint path. Should we resist the use of analogies, “there are only two courses open to us: either we must become first-rate mathematicians, able to live wholly among symbols and dispensing with pictorial images and such adventitious aid,” or—second option—we give up trying to grasp “the present state of electrical knowledge.”²³

This allusion to becoming a “first-rate mathematician” was no right-handed criticism, so to speak. Although Lodge himself could follow mathematical reasoning, he never contributed to the more formal side of physics in any way. He himself was among those who needed the analogies; they were addressed to him and other leading researchers like him, not just to the great mathematically unwashed.²⁴ Take Lodge's “cogwheel ether,” where the moving rack represented current and the gears set in motion stood for the ethereal whirls that constituted the magnetic field (Figure 2.5). Indeed, just because Lodge was willing to give up his understanding (his “grasp”) of modern electricity, he turned again and again to the illustrative-analogical: “Think of electrical phenomena as produced by an all-permeating liquid embedded in a jelly,” he says, “think of conductors as holes and pipes in this jelly, of an electrical machine as a pump, of charge as excess or defect, of attraction as due to strain, of discharge as bursting, of the discharge of a Leyden jar as a spring.”²⁵

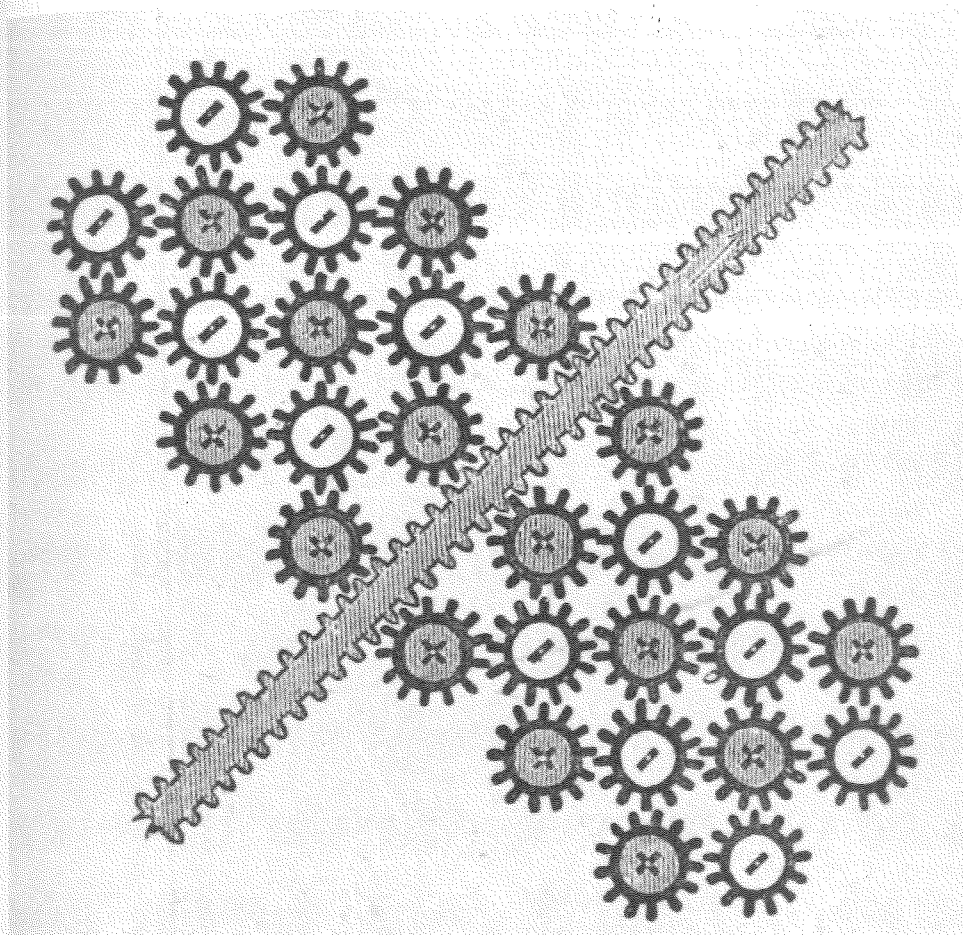


FIGURE 2.5. Oliver Lodge's cogwheel ether. The toothed bar, pulled through the apparatus of gears, is analogous to a current, and the cogwheels (labeled + and -) stand in for a magnetic field generated by the passage of that current. From Lodge, *Modern Views of Electricity* (London: Macmillan, 1907), 212.

Electricity could be grasped in terms of these mechanical concepts: strain, burst, jelly, gears, and whirls.

Lodge had in mind fully realized, *physical* models—he warned the reader to make sure the glass volume was fully filled, that all interfering bubbles had been duly pumped out (Figure 2.6). This “conspicuous,” manipulatable construction advanced understanding. By following the physical, the analogous Lodge explained to his readers, “you will have made a step in the direction of the truth, but I must beg you to understand that it is only a step; that what modifications and additions will have to be made to it before it becomes a complete theory of electricity I am unable fully to tell you. I am convinced they will be many, but I am also convinced that it is unwise to drift along among a host of complicated phenomena without guide

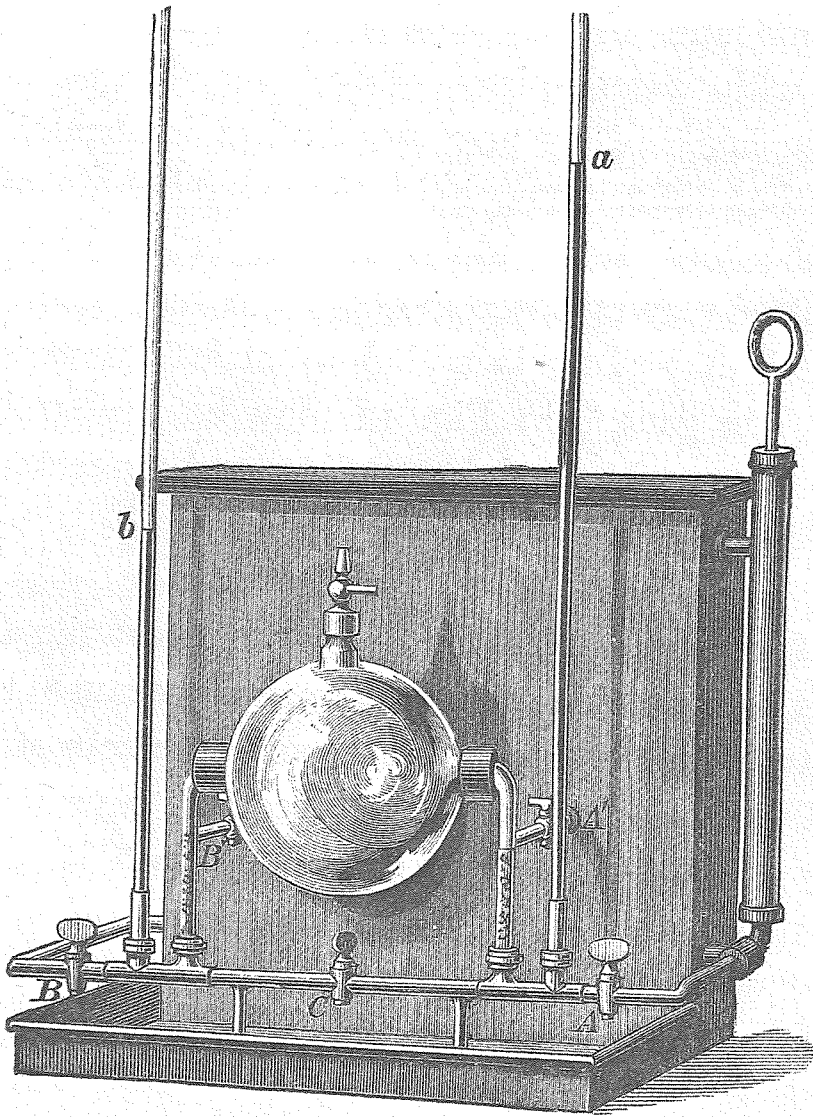


FIGURE 2.6. Oliver Lodge's fluidic-electricity; here a hydraulic model of a Leyden jar: electricity as water and gel. The whole, including the water gauges, is arranged vertically to be "more conspicuous." From Lodge, *Modern Views of Electricity* (London: Macmillan, 1907), 59.

other than that afforded by hard and rigid mathematical equations."²⁶ Models—physical, moving, material models—offered a path to a generative understanding that complemented the mathematical.

Laboring year after year, decade after decade, to capture the immaterial in the material, Lodge came ever more to see in the ether itself a form of substantiality.

In 1909, he issued an even more ambitious volume, *The Ether of Space*. As its frontispiece, he chose to depict the experimenter working from inside a massive analog device he dubbed “The Ether Machine.” By this time, he had come to think of the “insubstantial” ether to be anything but that. “I am able to advocate a view of the Ether which makes it not only uniformly present and all-pervading, but also massive and substantial beyond conception. It is turning out to be by far the most substantial thing—perhaps the only substantial thing—in the material universe. Compared to ether the densest matter, such as lead or gold, is a filmy gossamer structure; like a comet’s tail or a milky way, or like a salt in very dilute solution.”²⁷

Insubstantial—and most massive . . . beyond conception . . . the only substantial thing in the material universe. One thinks of Karl Marx’s evocative phrase “all that is solid melts into air.” Indeed, for Lodge, all that was material morphed into the immaterial, but conversely, all that had been immaterial had condensed into the most material stuff in all creation. In that back-and-forth, metaphorically, lies the evocative power of the material model. In the material model came a different, but coordinated, way of manifesting the mathematics.

AN ECONOMY OF WATER

Before World War II, New Zealander A. W. H. (Bill) Phillips had been a hands-on engineer trained in the electrical arts and worked at the remote Lake Waikaremoana hydroelectrical plant on the North Island—a bravura mix of electrical and hydraulic engineering.²⁸ When war broke out, he serviced airplanes, adjusting and modifying guns, among other things. After the war, Phillips headed to England for study at the London School of Economics. Facing lectures on economic theory, he found their abstract character “difficult to understand” and began sketching hydraulic relations. With Walter Newlyn, Phillips began thinking about the machine as a way of instantiating and making visible differential equations through a plumbing-based hydraulic model, in part to facilitate his own learning—an episode well documented (in much greater detail than is possible here) by historian of economics Mary Morgan.²⁹

Their ambitious project: to model the British economy with a connected set of water-carrying tubes, stopcocks, and reservoirs that could be controlled and adjusted. To build a device that could do this, Phillips and Newlyn bought war surplus hydraulic pumps originally destined to drive the landing gear, bomb bay doors, and aircraft fuel tanks. One pump had originally been designed for the reserve tank of a Spitfire. For the clear reservoirs, he cut Perspex (also known as Lucite or Plexiglass) sheets from the surplus windows of bombers (Figure 2.7).³⁰

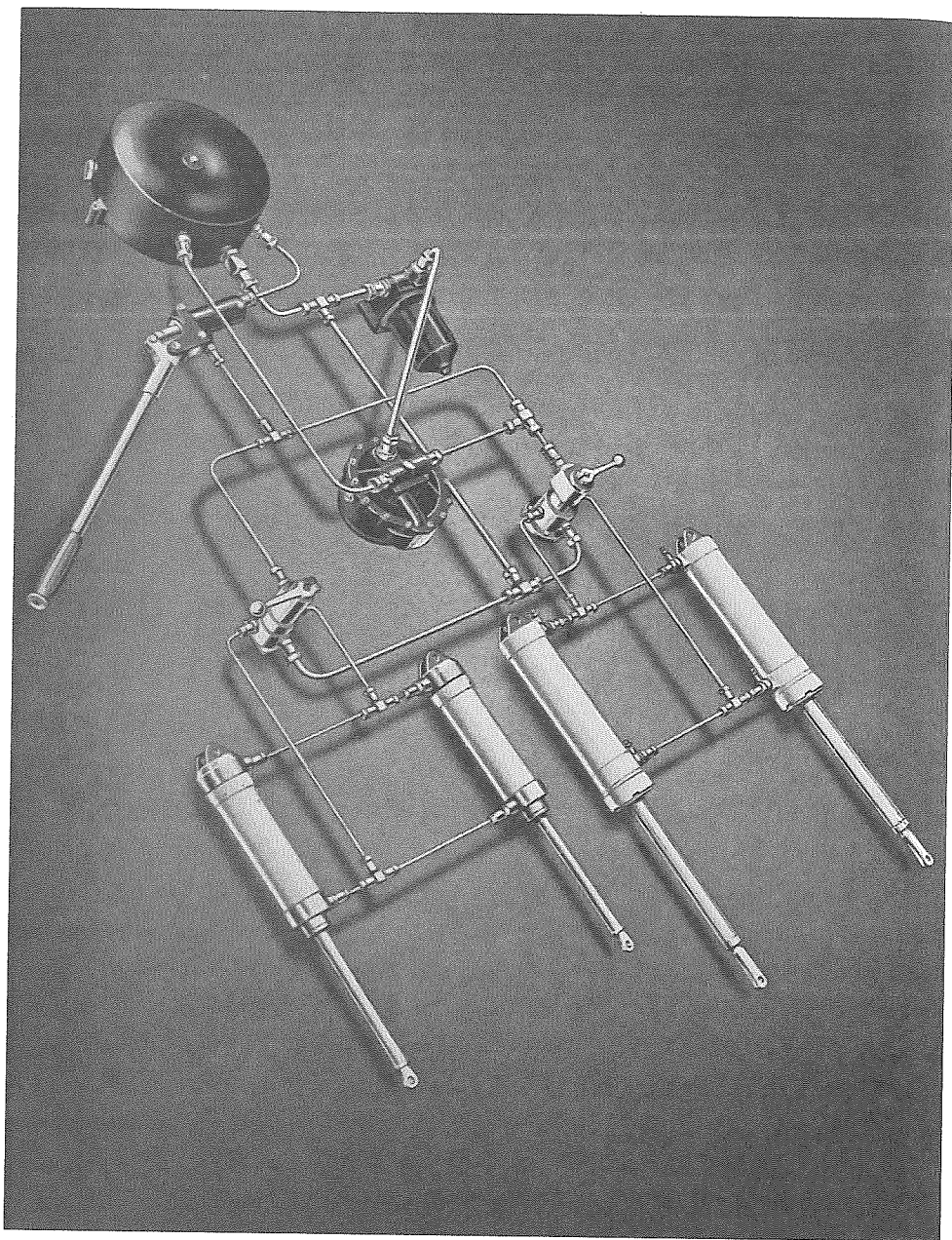


FIGURE 2.7. Dowty Equipment Ltd., instruction manual on aircraft hydraulics, including the Lancaster bomber undercarriage and bomb door, flap control unit, and emergency air shuttle valve. From <https://www.dowtyheritage.org.uk/wp-content/uploads/2019/02/Dowty-Manual-1940-2-658x494.jpg>.

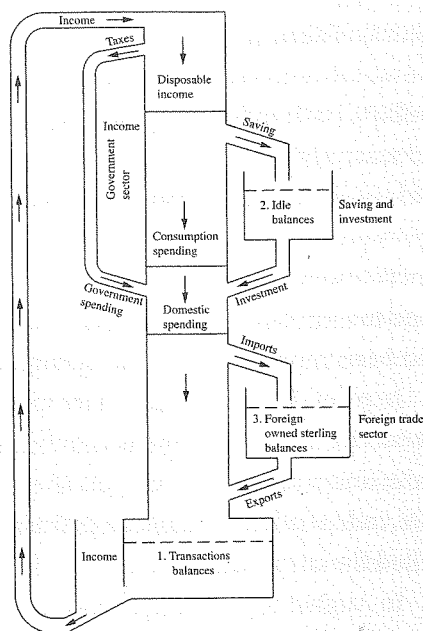
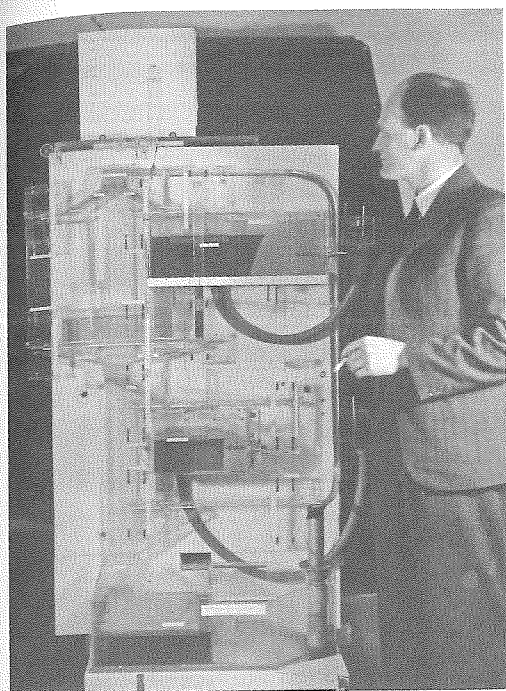


FIGURE 2.8. *Left*, Phillips and his machine—a photograph and a schematic. London School of Economics, circa 1950, https://www.sciencemuseum.org.uk/sites/default/files/styles/embedded_image/public/2018-12/Bill_Phillips_Machine_LSE.jpg. *Right*, a simplified view of the Phillips machine, from Nicholas Barr, “The Phillips Machine,” *LSE Quarterly* 2, no. 4 (1988): 321. Used with permission.

(The Lancaster bomber, from which Phillips was scavenging surplus materials, held a very complex hydraulic system, controlling landing gear, bomb bay doors, and more—with every part of the plane regularly subject to wartime stresses and battle damage.)

By November 1949, Phillips was ready to present his machine to the London School of Economics seminar. Seemingly against the odds, “both Phillips and the machine acquitted themselves well. Everyone who mattered was there. . . . Some [came] mainly to laugh. They gazed in wonder at this large ‘thing’ in the middle of the room” (Figure 2.8).³¹ Wonder indeed. Economic debates circulated around differential equations, data, and abstract models. The assembled regarded the pumping, gurgling machine with more than a little doubt. As for the speaker, “Phillips, chain-smoking, paced . . . explaining it in a heavy New Zealand drawl, in the process giving one of the best lectures on Keynes and Robertson . . . anyone . . . had heard. He then switched the machine on. And it worked! He really had created a machine that would simplify the problems and arguments economists had been having for years.” According to Keynes, the equilibrium interest rate would take on the value

that would induce people to hold the amount of money and number of bonds that were available. Robertson, by contrast, argued that the interest rate would be set by the supply and demand of loanable funds. The MONIAC, also referred to as the Phillips Hydraulic Computer and the Financephalograph, displayed visually and persuasively that the two formulations both held in equilibrium, water flowing in and out of the appropriate containers in steady measure.³² Water flow embodied the equations and ended the battle. As Mary Morgan put it well, “the Machine enabled a new understanding. . . . The system was genuinely dynamic—the liquid did take time to circulate. [It] deepened their understanding of the economic system that had been represented in the analogical model.”³³

In print, Keynes’s much more mathematical formulation and Robertson’s more discursive analysis for many economists floated past one another. Put into colored water and transparent Perspex, they could be understood, grasped without being able, independently, to reside entirely in the world of the equations. Flowing water stilled the dispute.

In August 1950, Phillips reported that “there has been an increasing use in economic theory of mathematical models, usually in the form of difference equations, sometimes of differential equations, for investigating the implications of systems of hypotheses. However, those students of economics who, like the present writer, are not expert mathematicians, often find some difficulty in handling these models effectively.” Phillips judged that the mechanical models played a crucial role by helping “non-mathematicians by enabling them to see the quantitative changes that occur in an inter-related system of variables following initial changes in one or more of them.”³⁴ A set of differential equations might capture these interrelated quantities, but *seeing* them could be crucial for an economist who was not simultaneously a mathematician—a professional not inside the charmed circle who lived and breathed in a purely symbolic world. In short, his aim to embody abstract relations mirrored those of Maxwell and Lodge, though eight decades later and in an entirely different discipline.

A few years later, Lionel Robbins, a leading member of the economics department at the London School of Economics, recalled the triumph of that moment: how the machine put into motion savings, investment, money supply, and liquidity. Suddenly embodied and controllable, “the subject of extensive debate for the preceding decade, resolved themselves almost automatically.”³⁵ Here was more than a pedagogical supplement. It was, instead, an advance, settling an ongoing abstract-economic debate in tactile-visible form.

Phillips looked at the machine as an alternate way of embodying the reality of a set of equations: the underlying mathematics was realized once in the actual flow of money, and at the same time in the flow of water: “fundamentally, the problem

is to design and build a machine the operations of which can be described by a particular system of equations which it may be found useful to set up as the hypotheses of a mathematical model, in other words, a calculating machine for solving differential equations." Other realizations were possible, to be sure. But, Phillips continued, "since . . . the machines are intended for exposition rather than accurate calculation, a second requirement is that the whole of the operations should be clearly visible and comprehensible to an onlooker."³⁶ With an electrical engineering background, certainly Phillips could have pursued a purely (analog) electrical computer (later in life, he did work with electrical analog computers). As a matter of prediction, it would certainly have been just as good, perhaps better, electrons being more controllable than water.

But instead, Phillips chose water flowing in Perspex. Like Maxwell and Lodge, Phillips turned to embodied abstraction to provide the viewer with the kind of kinesthetic access to the process itself—not just the prediction. Transparency and physicality mattered as another way into the problem: they mattered as a form of lateral grasp, to understanding.

BATHTUB BLACK HOLES

In 1986, Kip Thorne and Richard Price wrote a book on the "membrane paradigm" in black hole physics—an expression borrowed from Thomas Kuhn's use of *paradigm* in his *Structure of Scientific Revolutions* of 1962.³⁷ An older view (paradigm) in astrophysics, they argued, held black holes as "collapsed stars," a notion captured in Russian by the phrase "frozen star." That view peered at the horizon from far away—and from that reference frame, indeed, phenomena at the horizon appeared to "hover for all eternity." But, Thorne and Price wrote, a newer picture had emerged since the mid-1960s, taking seriously the notion of a co-moving, infalling frame of reference ("the black hole's point of view") in which one followed a particle (or observer) plunging past the horizon toward the singularity within.³⁸ Seen from this frame, the term *frozen* seemed fully out of place.

Strictly speaking, it should not matter which point of view one took; in the fullness of calculation, they should yield the same result. But, in fact, it does matter. Not just here but throughout theoretical physics (the authors insisted), "a special role is played by the diagrams, pictures, mental images, and descriptive phrases that accompany our equations—pictures of magnetic field lines threading through a conducting plasma and the corresponding phrase that the field lines are 'frozen into the plasma.'" Indeed, Thorne and Price contended that "a new set of pictures and descriptive phrases can have a profound impact on the subsequent development of a field of research." Suddenly, the new way of thinking let one imagine

the black hole horizon with an almost tangible physicality, as having conductivity, shear and viscosity, surface pressure, temperature, and entropy.³⁹

Thorne's view was that we need to be able to go back and forth between these points of view—he was *not* arguing that the membrane paradigm was more or less fundamental than the differential geometry with which it was associated; there was no vertical hierarchy. Conceptual dexterity, as Thorne saw it, opened up new ways of solving problems. The analogy structure, as so often was the case, made possible problem solutions in both directions.

One of the relativists who contributed importantly to the exploration of the close-in dynamics near the horizon was William Unruh. Unruh, a relativist at the University of British Columbia, asked just how much you can learn from looking at the world—specifically black holes—through analogies and, importantly, with fluid analogies. In 1981, Unruh published a paper in which he bemoaned the fragility of Hawking's work on black hole evaporation—and the difficulty of testing it on what would need to be tiny black holes in the vicinity of Earth. “However,” he continued, “a physical system exists which has all of the properties of a black hole as far as the quantum thermal radiation is concerned, but in which all of the basic physics is completely understood.”⁴⁰ Here again, precisely the Maxwellian ambition: a set of mathematical relations realized in two domains, one in the radically curved space-time of a black hole and one in the easily explored domain of the physical (fluid) analog. Much could be gained in the analog system, maybe even the detection of quantum phenomena.

Unruh later put it this way: “Obviously there are things that you can learn about the world [through analogies].” One can, he says, “also learn a lot about it by seeing how independent the phenomenon is of those differences, so that if you have areas in which you're very *different* than the original thing that you're looking at, then if the effect really is independent of that difference, you get much, much stronger faith that the effect is really going to be there.”⁴¹

For decades, many physicists considered black holes pathologies, as it were—unphysical or at least unrealized solutions to Einstein's eponymous equations. Then, in the early 1970s, astronomers detected material orbiting around an unseen gravitational source that indicated that the source might be so compact and so massive that it could not be made of any known kind of matter. In fact, there was no good explanation for what it could be other than a black hole. Black hole reality was reinforced at a much larger scale when stars could be seen dragged around the center of our galaxy—the Milky Way—in such a way and with such velocity that nothing but a supermassive black hole could be at work. More hints came from tremendous activity at the heart of many galaxies—perhaps these, too, were black holes, though much more massive. Then came LIGO's (Laser Interferometer

Gravitational-Wave Observatory, operated by Caltech and MIT) epochal gravity wave announcement of 2015—they had seen a signal of two black holes merging into a larger one. Bit by bit, these utterly unintuitive objects seemed to gain purchase on the physical world.

But how to understand them? Unruh often turned, surprisingly, if aptly, to Terry Pratchett's 1983 comic fantasy *The Color of Magic* where his imagined Discworld, among its many features, has an edge, a literal edge. Unruh, in a physics talk, found in that precipice an analog to the black hole: "to the fish, this Rimfall [the waterfall at the edge of the world disc] was a boundary, a horizon beyond which nothing could be heard. No fish who had ever travelled over the Rimfall had ever reported back. The shouts, or were they screams, of those intrepid explorers who had travelled too near to that boundary had suffered the most strange bass shifting, the fishes' high-pitched scream rapidly descending the scales to disappear from sound. Some claimed that with the most careful measurements of the sound, one could still hear sounds, of lower and lower frequency arbitrarily far into the future, as though the sound from those explorers never ceased. However, in no case could sounds ever be heard from beyond the location of that horizon, as that peculiar surface in the Rimfall came to be known."⁴² So it was with black holes. A luminous object falling toward and eventually through the horizon would be seen (from far away) as shifting in color ever more toward the orange (lower frequency), with no light ever received from beyond the (black hole) horizon.

To get a better sense of the physics, Unruh began developing a fluid mechanical model of what he dubbed "dumb holes." The fluid mechanical action for the system can be rewritten to show the mathematical correspondence between sound waves in a flowing fluid and scalar fields on a background space-time. The terms in the metric depend on the fluid velocity such that when the radial velocity of the fluid is equal to the speed of sound, there is a "horizon" analogous to the black hole event horizon. Unruh went on to outline various uses of this mathematical similarity. The first is that many calculations made by general relativists may be applied to problems in fluid dynamics. "One may also obtain insight into black holes through fluid experiments," Unruh commented. "Since surface waves in an incompressible fluid look like scalar fields on a metric, we can do 'black hole physics in a bath-tub.'" The analog of the space-time metric, as it turned out, could be controlled by the variation in the depth of the tub, while an adjustable drain set the rate of inward flow. Now one could look at the direct analog—in water and drain—of a wave scattering from the bathtub vortex and picking up energy. This was precisely the analog of superradiance, in which waves gained energy when they hit the entrained movement of space itself around a black hole (the ergosphere).⁴³

For both Maxwell and Lodge, electromagnetic phenomena could be demonstrated in the laboratory: their aim was to produce a parallel realization more accessible to the senses. Money could be tracked, but the machine could make manifest an analog process. But for these seeming sports of physics and astrophysics, the situation was different. Unruh: "We don't have little black holes around to . . . play with." But we do have whirlpools. "What [the whirlpool] illustrates is that the super radiant scattering is in fact very common. It's a ubiquitous phenomenon. It's not something that simply occurs for black holes, but occurs in everyday life. You get a big whirlpool in the ocean and there may well be situations in which the waves as they come in get amplified. What features are there of these kinds of systems that are going to produce super radiant scattering that are common to all of them."⁴⁴

For theorist-turned-experimenter Silke Weinfurter, there came a point when thought experiments, computer simulations, and analytic solutions were just not enough. In her words, the analog system of waves and whirlpools "is a completely different way of working. I'm a [theorist] by training, and . . . I thought I really understood that effect. I really understood it, I saw the derivations; I calculated myself. And then there's this real-life system and now everything has to be different. I have to rethink everything. How can I extract that?" So she did: after years of preparation and research, she came to run a "black hole laboratory" at the University of Nottingham, England. "There are . . . many layers of understanding something. And I always feel like the moment I've really understood an effect [is] when I've detected it." It "really changes the way you look at" the phenomena.⁴⁵

In such work, Weinfurter and Unruh joined the other figures we have discussed: none felt that the material models replaced the effects for which they are analogs; instead, the models dimensionalized understanding of the phenomena. As Weinfurter puts it, "there are some beautiful moments in an experiment, right? When something works out, you understand something. And [if] one of these things . . . we wanted to understand [is there] you will see that; it is pretty nice. You send these plane waves—a wave front towards the vortex—and then you get this bizarre, beautiful pattern which you can see by eye. . . . And you say, 'What is it actually? Where is this coming from?' And then you have to sit down and say, 'Okay, which theory? What is the main thing?' Is [the] pattern . . . just super radiance because you take some of the waves out on one side and you amplify the others? Does that give a pattern? And it turned out the pattern itself is just that [of a] wave . . . moving on a curved space time geometry. It's like light bending. . . . It's geometry becoming visible" (Figure 2.9).⁴⁶

Unruh underscored these fluid flow models. "If I could have a real black hole in my lab and I carried out this experiment [it would have been] very interest-

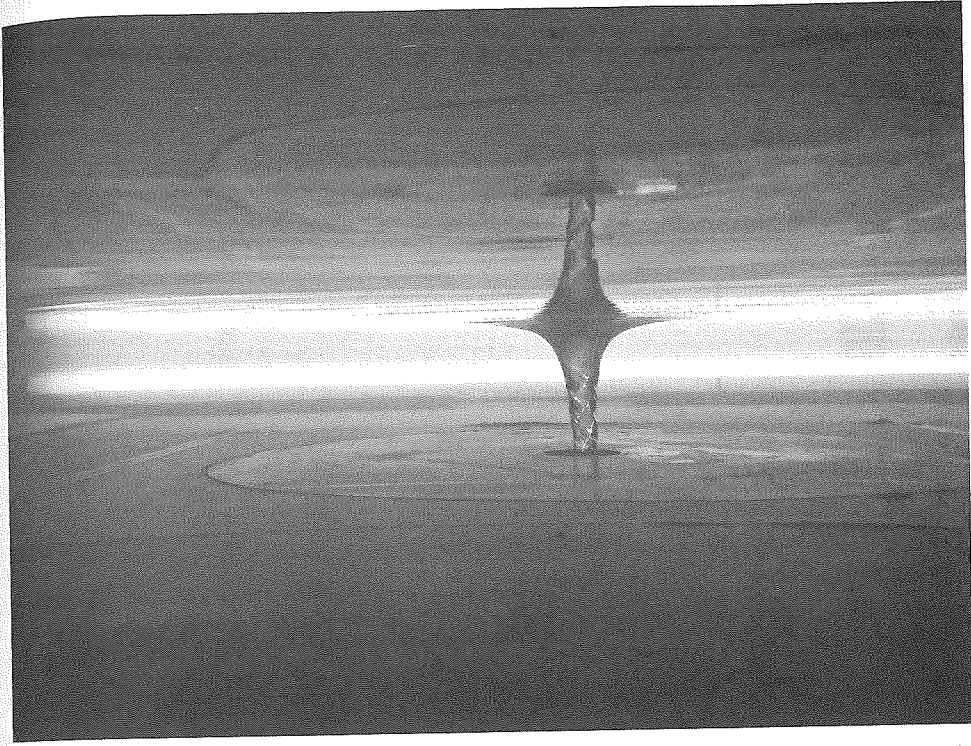


FIGURE 2.9. Bathtub black hole. Silke Weinfurtner's two-thousand-gallon "black hole laboratory" models many of the processes taking place just outside the horizon. Photograph by Peter Galison.

ing. . . . Carrying it out in another situation where we might think we understand the theoretical physics better than we do for black holes [as we do in the laboratory of surface waves and whirlpools] I think should have the same kind of weight as it would if one really carried it out for a black hole. It teaches us in fact even extra lessons for black holes. One could say okay, yeah, this happens for black holes. Who cares?" It seems rather unlikely that physicists might be so blasé, but the idea is clear: there is an underlying phenomenon captured by mathematical law and different ways of observing it, as in this diagram:

(black hole phenomena) ← (mathematical law) → (material analog)

Unruh, pressing the point, is saying that the law finds its realization and validation as much in an analog water wave setup as it does in light near a black hole. There is even the hope that the analog processes will lead to deeper waters, so to speak. One team, after contemplating the wide range of analog experiments, summed up their findings by saying that the analog physical models had been made to provide

“very down-to-earth models of otherwise subtle behavior of general relativity.” And yet there remained the speculative hope that “there may be more going on than just analogy—it is conceivable (though perhaps unlikely) that one or more of these analogue models could suggest a relatively simple . . . way of quantizing gravity that side-steps much of the technical machinery currently employed in such efforts.” This faith in the thoroughgoing power of the liquid analogy to Hawking radiation was by no means undisputed. But enthusiasts went so far as to see in the analog and quantum phenomena different versions of the *same* universal phenomenon.⁴⁷ Opening up parallel presentations of the structures held the possibility that vortices as well as equations held different ways forward.

From Silke Weinfurtner’s phrase “geometry becoming visible,” we have in compact form the aim of these material models. Laws of science may find their expression in equations; they may find their predictive expression through simulations; they may find their experimental expression in the laboratory. But embodiment is something else again: these are the instruments of understanding.

UNDERSTANDING/STANDING AMONG

Pondering the different modes of reasoning about science (algebraic, geometrical, material), Maxwell advocated an epistemic openness: “for the sake of persons of these different types, scientific truth should be presented in different forms, and should be regarded as equally scientific whether it appears in the robust form and the vivid colouring of a physical illustration, or in the tenuity and paleness of a symbolical expression.”⁴⁸ Acknowledging cognitive diversity has pedagogical consequences, as developmental psychologist Howard Gardner has insisted in a lifetime of work on “multiple intelligences.”⁴⁹ Children, like adults, learn in different ways and retain more effectively when they approach a subject in more than one mode—with figures and texts, for example. No doubt material models play a role in such strategies of teaching and learning, offering another path to grasping what to some learners might be obscure in the purely algebraic-mathematical, for example. Our focus here, however, has been less on learning than on Maxwell’s further claim, of greater import, that the modes he identified—symbolic, geometric, material—have equal claim to scientificity.

Indeed, Maxwell and our various other material modelers, from Oliver Lodge to Silke Weinfurtner, see phenomena manifested in different ways. What intrigues them is that something like superradiance can just as easily occur in the boosting of surface waves near a vortex as it can via light waves in the vicinity of spinning black holes. In both instances, waves can emerge with augmented energy. True,

we might study the effect more felicitously in a two-thousand-gallon tank of dyed water than in the ergosphere of a black hole, but rightly interpreted, so say our various witnesses, we are observing the same underlying effect. Unruh labels this recognition as one of universalizing—which is one way of putting the lateral correspondence stressed here.

No doubt the concatenation of different realizations of a phenomenon bolsters learning, sharpens the intelligibility of the phenomenon. But it does something more important still: it gives us a greater variety of ways to carry on, alternate paths to extend the research to a next theoretical or experimental step. If electrodynamic, economic, or black hole material models truly succeed, they do so by offering other paths forward with different affordances.

Do these material, mathematical, and visually related analogies *explain*? In certain cases, they may. But the lateral correspondences are not, I would argue, principally aimed at explanation, the way a vertical (reductive or unifying) relation might be. Exaggerating for impact, Unruh noted that we might be working intensively on a phenomenon (like superradiance) manifested in fluid flow and yet be barely interested in actual black holes. His logic is clear: there is a phenomenon that is widespread (say, superradiance), manifested in different physical systems (around bathtub vortices and black holes). To this we can add that in such situations, it just cannot be that black holes *explain* vortical fluid flow *and* vortical fluid flow *explains* black holes. But it surely can be that focusing on the whirlpools may lead us to insights that might have been harder to apprehend through the study of differential equations governing space-time geometry.

Much of modern physical science is now advancing through such linkages, better captured by lateral correspondence than by vertical reduction. Indeed, in many engineering contexts, such a turn has grown increasingly clear. Modern system engineers regularly learn to link electrodynamic and mechanical domains: the analogies (and there are many) signal a commonality of structure. But they are not at all trying to *explain electrodynamics*. What matters for designers and engineers is the ability to carry on, and the lateral (analogical) structures allow them to do just that. In mechanical engineering, over the course of the twentieth century, it became commonplace to invert the Maxwellians project and to use, analogically, reasoning about electrical circuits and their components to design *mechanical* systems (like mechanical frequency filters).

In fact, the engineering sciences show us a stance that captures a certain indifference toward ontology and a studied focus on unifying structure. Consider Ilene Busch-Vishniac, author of one of the definitive turn-of-the-twenty-first-century treatises on the myriad devices, known as transducers, that take one form of energy

and transform it to another.⁵⁰ Traditionally, transducers were thought to go from or to electrical domains: mechanical motion to/from electricity, heat to/from electricity, and so on. Her view is much less centered, no longer privileging voltages and currents. Instead, she sees transducers as embracing the full set of transformations in and out of chemical, electrical, thermal, mechanical, and other forms of energy. Some might try to partition optical, electrical, and magnetic processes; others prefer to group them together, because they all can be considered manifestations of Maxwell's theory. For Busch-Vishniac, such debates seem irrelevant. Instead, her interest is in the general set of phenomena captured by transduction: rotatory motion to electrical signals; magnetic fields to heat; indeed, the whole highly diverse set of transformations (every possible conversion between chemical, magnetic, thermal, optical, mechanical, and electrical), including transformations within a domain (e.g., magnetic to magnetic). To develop new sensors and actuators is to get to the business of calculating, designing, and realizing—with studied indifference to ontologies:

While these philosophical debates are interesting, they are moot from the perspective of understanding transduction, because the focus is on conversion of energy in a device by such a means as to make it useful in monitoring a system or imposing a state on a system [sensing or transducing]. Indeed, the modeling technique that will be used in this book has been applied to many different energy domains, and to various manifestations of energy in the same broad domain with virtually no change required by shifting of the energy types.⁵¹

In designs of hearing aids, nail guns, vacuum cleaners, the back-and-forth between mechanics and electrical theory is constant. Analogies are tools to grasp relations and to build things. There are choices about how to set up these correspondences (between, say, springs and capacitors; dashpots [damped plungers] and resistors; and masses and inductors). But the goal is making.

Even if contemporary engineers extend the linkages of analogies far beyond where Maxwell took them, the nineteenth century, the spirit of Maxwell's analogizing holds fast. Indeed, in 1998, when Busch-Vishniac looked to ground her choice of the analogical structure, she chose the one called the Maxwell or impedance analogy (with mechanical, electrical, and fluid impedance forming a three-way correspondence). In the midst of a full range of late twentieth-century references, she cited one and only one much older work, James Clerk Maxwell's 1865 essay "The Dynamical Theory of the Electromagnetic Field."

If we wanted a slogan, it would be *correspondence without hierarchy*. Not a unification under a single set of entities or laws but instead an alignment of

approaches, making common cause in understanding aspects of the world. When one is building so many forms of electromechanical, thermomechanical, or electro-optical devices, the lateral connections make reasoning vastly easier. Such a stress on lateral modeling has become ever more prevalent, and not just in engineering. One of the greatest contributions to theoretical physics in recent years is known as AdS/CFT, linking a form of curved space-time, anti-de Sitter space (an oft-used form of space-time geometry in gravitational theory), to conformal field theory (the language of particle physics). Some theorists have used AdS/CFT to extend insight *from* quantum field theory *into* gravity; others use the correspondence to extend insight *from* gravity *into* the quantum field theory of heavy ions (for example). Such cross-links expand our circle of understanding and mark much of the most intriguing tendencies in physics today.

There is an expanding form of unity within the physical sciences and engineering, but it is not the unity of laddered ascent (or descent from a single law or set of entities down through sciences considered derivative). Nor is that unity one of homogenization, just as electrodynamics does not *become* mechanics. Instead, distinct approaches with distinct ontologies find concordances—dualities, symmetries, corresponding structures. We are faced with a ring, linked profoundly in its parts but without a consensual center or peak: a ring, not a pyramid.

True, we live in an age when, to account for black holes, physics draws on abstract field theory, artificial intelligence, Monte Carlo simulations, topology, geometry, sensitive laser interferometers, and extended telescope arrays. And yet we can still work to find a way forward linking these directions—in the fineness of differential geometric abstraction and in the bulk of a two-thousand-gallon tank of swirling green water. Standing amid them all, tracing the lines of connection, perhaps we can find understanding.

NOTES

I would like to thank the editors of this volume for very helpful comments along the way and Erik Curiel, Jordi Cat, and members of the philosophy seminar within the Harvard Black Hole Initiative for precise and thoughtful suggestions. Appreciation, too, to Nicole Terrien for research and help with manuscript preparation. This project is funded in part by the Gordon and Betty Moore Foundation. It was also made possible through the support of a grant from the John Templeton Foundation. Opinions expressed in this publication are those of the author(s) and do not necessarily reflect the views of the John Templeton Foundation.

- 1 Catherine Elgin argues that the “felicitous falsehoods” of such idealizations exemplify facts, even though the idealizations are, strictly speaking, false—and that this process of exemplification delivers a form of understanding. Elgin, “Understanding and the Facts,” *Philosophical Studies* 132 (2007): 33–42.
- 2 There is a quasi-infinite literature on modeling in the sciences, but a recent collection offers a good launching point with some superb essays addressing periods from the eighteenth century forward: Soraya de Chadarevian and Nick Hopwood, eds., *Models: The Third Dimension of Science* (Stanford, Calif.: Stanford University Press, 2004).
- 3 Jonathan Kvanvig takes understanding to be an “honorific”—that is, understanding serves as nothing more than a compliment, not a justified epistemological state of affairs. See Kvanvig, “Knowledge and Understanding,” in *The Value of Knowledge and the Pursuit of Understanding*, 185–203 (Cambridge: Cambridge University Press, 2003). This demotion of the concept of understanding has a long history in recent decades. Carl Hempel, in *Aspects of Scientific Explanation and Other Essays in the Philosophy of Science* (New York: Free Press, 1965), 413, clearly considered understanding to be nothing more than a psychological trait, not part of the knowledge structure itself. That line of assessment has continued, as Henk W. de Regt and Dennis Dieks rightly contend, in work by Bas van Fraassen and J. D. Trout—see their “A Contextual Approach to Scientific Understanding,” *Synthese* 144 (2005): 137–70, esp. 141.
- 4 Michael Friedman, “Explanation and Scientific Understanding,” *Journal of Philosophy* 71 (1974): 18.
- 5 Henk W. de Regt, *Understanding Scientific Understanding* (Oxford: Oxford University Press, 2017), 12. My emphasis here is more on understanding phenomena than theories per se. I very much appreciate Howard Stein’s formulation of understanding as fundamentally joined to the clarification of concepts. “The Enterprise of Understanding and the Enterprise of Knowledge,” *Synthese* 140 (2004): 135–76.
- 6 Jordi Cat, “On Understanding: Maxwell on the Methods of Illustration and Sci-

- entific Metaphor,” *Studies in History and Philosophy of Science, Part B: Studies in History and Philosophy of Modern Physics* 32, no. 3 (2001): 436.
- 7 On forms of coordination, trading zones, and scientific interlanguages, see Galison, *Image and Logic: A Material Culture of Microphysics* (Chicago: University of Chicago Press, 1997).
- 8 *Online Etymology Dictionary*, s.v. “understand,” <https://www.etymonline.com/word/understand>.
- 9 For more on analog models across a wide range of fields, see Susan G. Sterrett, “Experimentation on Analogue Models,” chapter 39 in *Springer Handbook of Model-Based Science*, ed. Lorenzo Magnani and Tommaso Bertolotti, (New York: Springer, 2017), and further references therein. Sterrett rightly points out how widespread the analog models are and have been—and that digital simulations often must contain vastly more information than “similarity models.”
- 10 On the vertical (pyramid) versus lateral (ring) forms of linkages among theories, see Peter Galison, “The Pyramid and the Ring: A Physics Indifferent to Ontology,” in *Research Objects in Their Technological Setting*, ed. Bernadette Bensaude Vincent, Sacha Loeve, Alfred Nordmann, and Astrid Schwarz, 15–26 (New York: Routledge, 2017), and his “Meanings of Scientific Unity: The Law, the Orchestra, the Pyramid, Quilt, and Ring,” in *Pursuing the Unity of Science: Ideology and Scientific Practice from the Great War to the Cold War*, ed. Harmke Kamminga and Geert Somsen, 12–29 (Burlington, Vt.: Ashgate, 2016).
- 11 James Clerk Maxwell, “A Dynamical Theory of the Electromagnetic Field, 1864,” in *The Scientific Papers of James Clerk Maxwell*, ed. W. D. Niven (New York: Dover, 1965), 1:563–64. It is exactly this contrast between understanding and explanation that I find intriguing, and not because understanding it is *merely* illustrative; not at all—instead, understanding emerges from the complementarity of different realizations of a theory. Emphasis on “understanding” added in the Maxwell quotation.
- 12 James Clerk Maxwell, “Address to the Mathematical and Physical Sections of the British Association,” in Niven, *Scientific Papers*, 2:219, hereinafter Maxwell, “Presidential Address.” The literature on models in Maxwell and Maxwellian electrodynamics is vast. Intrinsically outstanding and of use for further reference are, for example, Jed Z. Buchwald, *From Maxwell to Microphysics: Aspects of Electromagnetic Theory in the Last Quarter of the Nineteenth Century* (Chicago: University of Chicago Press, 1985), which makes clear that the ether models became a huge difficulty later when faced with Lorentz’s work toward a particle and field picture of microphysics, and Daniel Siegel, “The Origin of the Displacement Current,” *Historical Studies in the Physical Sciences* 17 (1986), 99–146; for a very perspicuous synthetic work on electrodynamics including the role of models for Maxwell in his electrodynamics, see Olivier Darrigol, *Electrodynamics from Ampère to Einstein* (Oxford: Oxford University Press, 2000), esp. 147–63. See

- also Cat, "On Understanding." He contextualizes many of Maxwell's views in the philosophical psychology of Victorian England and stresses, rightly in my view, that the kind of understanding he sees as a goal of Maxwell's work is not fundamentally tied to Maxwell's idea of explanation.
- 13 Maxwell, "Presidential Address," 219–20.
 - 14 Maxwell, 220.
 - 15 Maxwell, 220.
 - 16 James Clerk Maxwell, "On a Dynamical Top, for exhibiting the phenomena of the motions of a body of invariable form about a fixed point, with some suggestions as to the Earth's motion," in Niven, *Scientific Papers of James Clerk Maxwell*, 1:248.
 - 17 Sigmund Freud, "Creative Writers and Day-Dreaming," *Standard Edition* 9 (1959): 142–53, "date-mark" on 147.
 - 18 Maxwell, "On Potential," in *The Scientific Letters and Papers of James Clerk Maxwell*, ed. P. M. Harman (Cambridge: Cambridge University Press, 1990), 3:787, emphasis added.
 - 19 Maxwell, "Presidential Address," 219. Maxwell more extensively treats the idea of classification in his 1869 "On the Mathematical Classification of Physical Quantities," in *The Scientific Papers of James Clerk Maxwell*, ed. W. D. Niven (New York: Dover, 1965): 257–66. Here Maxwell distinguishes the formal classification from the physical classification. Thanks to Erik Curiel for discussion of this paper.
 - 20 Maxwell, 226–27.
 - 21 Maxwell, 227.
 - 22 Maxwell, 219. Bruce Hunt on "broadening" (with which I completely agree), *The Maxwellians* (New York: Cornell University Press, 1994), 74, though presenting this as broadening the paths to "truth" may be overstated.
 - 23 Oliver Lodge, *Modern Views of Electricity* (London: Macmillan, 1907), 12–13.
 - 24 Lodge, 212. See also Hunt, *Maxwellians*, 89–93, for an excellent discussion of this and similar mechanical model figures.
 - 25 Lodge, *Modern Views*, 60–61. See also Hunt, *Maxwellians*, 90–91.
 - 26 Lodge, *Modern Views*, 60.
 - 27 Oliver Lodge, *The Ether of Space* (New York: Harper, 1909), xviii.
 - 28 Alan Bollard, *A Few Hares to Chase: The Economic Life and Times of Bill Phillips* (Oxford: Oxford University Press, 2006), 28ff.
 - 29 Phillips wrote about this to Richard Sayers, in Bollard, *A Few Hares to Chase*, 98–99; Mary Morgan, *The World in the Model: How Economists Work and Think* (Cambridge: Cambridge University Press, 2012), esp. chapter 5 (172–216), where she addresses the Phillips–Newlyn Machine with great clarity, justly evoking Newlyn's contribution in much greater depth than I can here.
 - 30 Bollard, *A Few Hares to Chase*, 101.
 - 31 Nicholas Barr, "The History of the Phillips Machine," in *A. W. H. Phillips: Collected Works in Contemporary Perspective*, ed. Robert Leeson (Cambridge: Cambridge University Press, 2000), 92.

- 32 Nicholas Barr, "The Phillips Machine," *LSE Quarterly* 2 (1988): 310, discussion of Robertson and Keynes on 329.
- 33 Morgan, *World in the Model*, 210. A good summary of the dispute can be found in Andres Nentjes, "The Keynes versus Robertson Controversy in Monetary Economics with a Post-Scriptum on the Dutch Monetary Controversy in the Nineteen Fifties," *De Economist* 127, no. 4 (1979): 551–69.
- 34 A. W. H. Phillips, "Mechanical Models in Economic Dynamics," *Economica*, New Series 17, no. 67 (August 1950): 283.
- 35 Lionel Robbins recalling the demonstration from November 1949, quoted in Bollard, *A Few Hares to Chase*, 108.
- 36 Phillips, "Mechanical Models in Economic Dynamics," 283.
- 37 Thomas Kuhn, *The Structure of Scientific Revolutions* (Chicago: University of Chicago Press, 1962).
- 38 Richard Price and Kip S. Thorne, introduction to *Black Holes: The Membrane Paradigm* (New Haven, Conn.: Yale University Press, 1986), 1.
- 39 Price and Thorne, 1. The geometrical approach advocated by Price and Thorne draws deeply on the geometrical (geometro-dynamical) approach advocated by Charles Misner, Kip Thorne, and John Wheeler in their generation-marking textbook *Gravitation* (Princeton, N.J.: Princeton University Press, 1973).
- 40 Unruh goes on to sketch the lines of a possible inquiry: "In this system one can investigate the effect of the reaction of the quantum field on its own mode of propagation, one can see what the implications are of the breakdown of the wave equation at small scales on the evaporation process, and one might even contemplate the experimental investigation of the thermal emission process." William G. Unruh, "Experimental Black-Hole Evaporation?," *Physical Review Letters* 46 (1981): 1351.
- 41 William G. Unruh, interview with Peter Galison, Cambridge, Mass., April 23, 2018, hereinafter, Unruh and Galison, interview.
- 42 William G. Unruh and Ralf Schützhold, eds., *Quantum Analogues: From Phase Transitions to Black Holes and Cosmology* (Berlin: Springer, 2007), 1.
- 43 Matthew Hasselfield, "A Graduate Student Summary of the Public Lecture 'Black Holes/Dumb Holes: Condensed Matter Analogues' by Bill Unruh," PITP Showcase Conference, May 2005, <https://pitp.phas.ubc.ca/archives/CWSS/showcase/unruhsum.html>. "Hawking radiation seems to give much insight into black hole physics, in particular by providing a framework for black hole thermodynamics. The thermodynamical justification for Hawking radiation is so strong that Unruh feels that 'this thermodynamic analogy makes one believe that the result is right even though the derivation is nonsense.'"
- 44 Unruh and Galison, interview.
- 45 Silke Weinfurtner, interview with Peter Galison, Cambridge, Mass., May 9, 2018, hereinafter Weinfurtner and Galison, interview.
- 46 Weinfurtner and Galison, interview. Also a strong counterargument to the force of

- analogue experiments on Hawking radiation: Karen Crowther, Niels Linnemann, and Christian Wuthrich, "What We Cannot Learn from Analogue Experiments," *S.I.: Reasoning in Physics*, 2019, <https://link.springer.com/article/10.1007/s11229-019-02190-0>. My own view is that the analog experiments work well in addressing superradiance, which is not "as inaccessible" (in several ways) as Hawking radiation.
- 47 Carlos Barceló, Stefano Liberati, and Matt Visser, "Analogue Gravity," ArXiv:gr-qc/0505065v3 (May 12, 2011), 108. For a discussion of the philosophical views in favor of and against the analog models of Hawking radiation, see Erik Curiel, "Singularities and Black Holes," in *The Stanford Encyclopedia of Philosophy*, Spring 2021 ed., ed. Edward N. Zalta, <https://plato.stanford.edu/archives/spr2021/entries/spacetime-singularities/>.
- 48 Maxwell, "Presidential Address," 220.
- 49 Gardner's first major presentation of these ideas was in Howard Gardner, *Frames of Mind: The Theory of Multiple Intelligences* (New York: Basic Books, 1983) and then was developed and debated in many contexts among educators and psychologists.
- 50 Ilene J. Busch-Vishniac, *Electromechanical Actuators and Sensors* (New York: Springer, 1998), 2–4.
- 51 Busch-Vishniac, 5.